European
Historical
Economics
Society

EHES WORKING PAPERS IN ECONOMIC HISTORY | NO. 6

Communal Responsibility and the Coexistence of Money and Credit under Anonymous Matching

Lars Boerner,
Free University of Berlin

Albrecht Ritschl,
LSE and CEPR

JANUARY 2011

EHES Working Paper | No. 6 | January 2011

Communal Responsibility and the Coexistence of Money and Credit under Anonymous Matching

Lars Boerner,
Free University of Berlin

Albrecht Ritschl,
LSE and CEPR

First draft: June 2006
This version: December 2010

Abstract

Communal responsibility, a medieval institution studied by Greif (2006), supported the use of credit among European merchants in the absence of modern enforcement technologies. This paper shows how this mechanism helps to overcome enforcement problems in anonymous buyer/seller transactions. In a village economy version of the Lagos and Wright (2005) model, agents trading anonymously in decentralized markets can be identified by their citizenship and thus be held liable for each other. Enforceability within each village's centralized afternoon market ensures collateralization of credit in decentralized markets. In the resulting equilibrium, money and credit coexist in decentralized markets if the use of credit is costly. Our analysis easily extends itself to other payment systems like credit cards that provide a group identity to otherwise anonymous agents.

JEL codes: E41, D51, N2

Keywords: Communal responsibility, anonymous matching, money demand, credit, bills of exchange

Acknowledgements:
Helpful comments by Avner Greif, Belén Jerez, Gary Richardson, Harald Uhlig, Randall Wright, and participants in various seminars and workshops are gratefully acknowledged. This research was supported by Deutsche Forschungsgemeinschaft under collaborative research center SFB 649. Contact: Lars.Boerner@fu-berlin.de; a.o.ritschl@lse.ac.uk.

Communal Responsibility and the Coexistence of Money and Credit Under Anonymous Matching

1 Introduction

How can the use of credit in anonymous exchange be supported in equilibrium? Does history offer examples of institutions that accomplish this? Financial instruments only arise when trust or enforcement are strong enough to break the one-shot character of anonymous barter and cash transactions. Before the advent of the modern territorial state with its law enforcement tools, traders could hardly hope for contract enforcement by authorities, whose reach was incomplete and at best only local. Under those circumstances, transactions that were asynchronous across space and time were ruled out, or limited to small minorities where group cohesion afforded the necessary level of enforcement, see Greif (1993).

According to conventional wisdom, such as van der Wee (1963) and North (1981), the fragmentation of jurisdictions across the many micro-territories of Continental Europe acted as an obstacle to enforceability and long prevented the rise of financial instruments. In this perspective, territorial unification in the late 13th century, which provided England and France with unified jurisdictions, enhanced contract enforceability and gave both countries a decisive advantage in the development of financial institutions.

Recent research by Greif (2002), Greif (2006) has qualified this picture. Drawing on extensive research by legal historians, Greif analyzed the medieval institution of merchants' communal responsibility, by which merchants from a given city would be held collectively liable for each other's debts when trading abroad. By agreements between their jurisdictions, agents would have access to each other's city courts to settle their claims individually, and communal responsibility would serve as a means of collateralizing any debts in case of dispute. Greif shows that equilibria may exist under which agents report their claims truthfully if a case is brought to court.

In this paper, we build on this result and extend it both historically and theoretically. In the case of England studied by Greif (2002, 2006), communal responsibility was only a short-lived institution, as the transition to individual responsibility under the unified jurisdiction of a territorial state was made quickly. In continental Europe, communal responsibility survived much longer. Boerner and Ritschl (2002) looked into cases from Germany and Italy where territorial unification was absent, and found that communal responsibility was highly persistent and evolved into an elaborate system of trade treaties. In the sequel, we briefly review additional evidence to suggest that during centuries, communal responsibility was a collateralization technology supported the use of financial instruments in places where territorial integration came late.

On a theoretical level, we examine the implications of communal responsibility as a collateralization device in a random matching model of monetary demand. Research in the tradition of Kiyotaki and Wright (1989) and Kiyotaki and Wright (1991) has studied the use of money in a matching framework, in which buyers and sellers match randomly for anonymous, one-shot exchanges. Anonymity in such a setup can be interpreted as traders' payment and credit history being unobservable, see Kocherlakota (1998). We adopt the canonical setup of Lagos and Wright (2005), which includes two consecutive markets per period, one decentralized and anonymous, the other centralized and Walrasian. In the standard setup, anonymity prevents agents from using credit in the decentralized market, making the use of money essential. In contrast, claims in the centralized market are fully enforceable.

Introducing credit in these matching models has proven to be difficult. He, Huang, and Wright (2005) devise a matching model in which money and credit coexist. In their model, money is indivisible, giving rise to credit transactions. Berentsen, Camera, and Waller (2007) introduce banking into the Lagos and Wright (2005) model by assuming that banks can observe agents' financial histories, but not their trading histories. All trade transactions in the decentralized matching market are then still cash-based, but traders can borrow money from banks before the transaction. However, as agents need to avail themselves of cash before the match outcomes are revealed, there still exists a money holdup in the decentralized market. Telyukova and Wright (2008) introduce a sequence of alternating decentralized and centralized markets per period. There, credit is generated in the Walrasian centralized markets but not used in the decentralized matching market. In Lagos and Rocheteau (2008), capital goods compete with money as a medium of exchange in the decentralized market, and there exists an incentive for agents to overproduce capital goods in order to have a means of payment if fiat money is not available. Martin, Orlando and Skeie (2006) devise a model of credit in the decentralized market, in which both sides to a transaction have access to a machine providing individual credit histories before concluding a deal. Technically, our approach is very similar to this, except that we do not need individual credit histories but only the collective ones, and that we have collective enforcement. A merchant from a city whose record is in good standing will obtain credit, and communal responsibility implies that the merchant's fellow citizens will become the seller's enforcement agents.

To implement this idea, we follow Faig (2004) and Faig and Jerez (2006), who interpret the matching setup of the Lagos and Wright (2005) model as a village economy. In their model, cash is used for transactions in anonymous matching markets across villages, and credit for transactions within villages. In Faig (2004) and Faig and Jerez (2005), however, all credit is again in the centralized market, and serves purely for financial and insurance motives. Our theoretical contribution will be to implant credit into the decentralized matching market that governs transactions across villages.

With the availability of collateralized credit in the decentralized market, coexistence of money and credit becomes an issue: is money still useful or welfare-improving if credit is available? As it is costly to frontload cash, credit would drive cash out of the market whenever the Friedman rule does not apply. On the other hand, credit is not costless either. In the historical cases we study, credit letters were either notarized or registered with the city court, which entailed nontrivial cost. Models with ad hoc cash in advance restrictions in which there are cost thresholds to the use of credit are presented e.g. by Prescott (1987), Freeman and Huffman (1991), Ireland (1995), Chatterjee and Corbae (1996), Lacker and Schreft (1996), and Cavalcanti and Wallace (1999). We borrow from this literature and assume that the use of credit carries a fixed resource cost.

The choice between cash and credit in a matching model becomes nontrivial if match outcomes are risky. In line with the Lagos and Wright (2005) model, we assume that buyers face preference shocks, which follow a two-state Markov chain and are only revealed once the match is established. Then, bargaining outcomes are Markov as well, and buyers face the problem of optimally choosing their cash demand. We find that if the cost of credit is positive but not prohibitively high, agents will hold cash to pay for small transactions, and ride out positive demand shocks by using credit.

The rest of this paper is organized as follows. Section 2 briefly reviews evidence on the history of credit instruments in Europe and puts their evolution in the context of communal responsibility. Section 3 adapts the Lagos and Wright (2005) framework. Section 4 sketches conditions for the coexistence of money and credit in an anonymous matching market. Section 5 characterizes social planner and decentralized market solutions with and without communal responsibility. Section 6 discusses the results and places them in the context of modeling alternatives and their possible historical interpretations, tracking historical analogs to modeling devices that have become popular in the matching literature, such as the big family assumption of Shi (1997). Section 7 concludes with implications for further research.

2 Historical Background

To remit money from one place to another, 13th century Italians developed the industry of "campsores", money changers who would receive cash payment in one location and pay out the equivalent in a different location. The term for these money remittances, "cambium" (change), soon became synonymous for a wider class of payment transactions, in which credit was provided, see Freundt (1899). Along the Mediterranean, remittance and credit contracts were often notarized, and two parallel letters of credit issued to be sent by separate mail, to make the presenter's claim more credible. In Western and Central Europe, court registers would soon assume the same task, so that the second letter fell out of use. In contrast to the later bills of exchange, these early credit letters were not yet a negotiable title that constituted an enforceable property right. Legal scholars in the tradition of the Historical School argued over the origins of the bill of exchange, see Goldschmidt (1891), Schaps (1892), Schaube (1898), Freundt (1899); for a brief summary of the argument and rich additional evidence see de Roover (1953). These early researchers agreed that money transfer and credit operations across cities occurred early, often, and were regularly brought to and enforced by the municipal courts.

When these assets became tradable has continued to be a matter of dispute. Usher (1943), van der Wee (1963), and Optiz (1968) argued for a rather late beginning, citing court cases from Flanders from the early 16th century. From then on, the holder of a bill of exchange was entitled to payment, no matter what the underlying original creditor/debtor relation was. de Roover (1953) finds tradability of bills of exchange in Italy to be a rare exception before the 16th century. Munro (1991), Munro (1994) argues that tradability of bills of exchange was the norm under the Law Merchant (albeit not under the Common Law) in medieval England already in the 14th century. However, bills of exchange were apparently used predominantly in England's foreign trade, especially with the Low Countries. For domestic transactions, easier, seemingly less sophisticated instruments were in use. Comparatively little attention has been paid to developments in Germany. Recent research by Rothmann (1998) has filled the gap for the city of Frankfurt, partly by consulting archival sources from Frankfurt's trade partners, often smaller cities throughout the German-speaking countries whose records survived World War II. Rothmann cites evidence for the use of bills of exchange, and their transfer from one party to the other, for as early as the late 14th century. In the first half of the 15th century, bills of exchange payable in Frankfurt regularly included the clause ”wer den brief innhat” (who the bearer of the letter is) (Rothmann, p. 335, p. 484). This is remarkably close to similar stipulations on English credit letters issued under the Law Merchant at the same time (see Munro, 1991). Boerner and Hatfield (2010) show that from the Late Middle Ages onwards complementary institutions in form of decentralized organized clearing houses in fair places in West and Central Europe evolved which made efficient clearing possible without using transfereable bills.

Like elsewhere in Europe (on the prominent cases of the Champagne fairs, later Geneva, Lyon, and Genua, see Boyer-Xambeu, Deleplace, and Gillard (1991)), Frankfurt's trade fairs soon became nationwide clearing institutions. For a sample of cities from Germany and Switzerland, Rothmann documents that among the bills of exchange, mortgaged rents, and other credit and payment instruments registered in the respective city courts, those payable in Frankfurt or at the time of the Frankfurt fairs are dominant. Thus, the fairs synchronized and integrated financial markets in all of Southern and Central Germany, even for payments that were not physically due in Frankfurt itself. Apparently, financial instruments were well developed in most areas of Europe in the 15th century, and agents were able to pay without cash, and to transform one form of financial asset into another both over time and space.

Credit letters and other financial instruments like fixed-income rents critically depended for their practical use on enforceability. Notarization of credit letters was the norm in Southern Europe, see Freundt (1899), de Roover (1953). In central and Western Europe, registration in court or city registers prevailed, see Munro (1991). City courts convened to enforce payment, or to reach a binding settlement between both parties, and their rulings were apparently effective enough to secure the continuing use of financial instruments. The question is why this is.

The easy case to explain is medieval England. Under English Common Law, merchants registering their claim in the courts of York, London, or Bristol would have the right to seize the debtors' assets in all of England in the case of default (Munro, 1991). The enforceability of claims across a large, unified jurisdiction was undoubtedly a major advantage for England's commercial development (Greif, 2002, 2006). Beginning in the mid-14th century, enforceability of European-style law merchant claims was assured by giving merchant courts applying these rules the right to seize assets. Again, this gave England an advantage, and Munro (1991, 1994) argues that the Law Merchant gradually crowded out the more expensive Common Law debt instruments among English merchants.

Merchants on the European continent had to cope with fragmented jurisdictions. Feuds, wars, and city bankruptcies inhibited the flow of goods and capital across locations . Still, financial flows were surprisingly undisturbed by such events, and the limitation of local jurisdictions hardly seems to have been a problem. In 1410, the city of Cologne banned its citizens from attending the Frankfurt fairs in order to force Frankfurt to lower its fees. Only those having to settle their debts or collecting debt in Frankfurt were allowed to go (Rothmann, 1997, p. 422). While debt default by institutional debtors, such as cities or territories, was a problem, individual default was not, at least no more than within a given jurisdiction.

The reason for this was the threat of communal responsibility. Under this seemingly archaic institution, merchants would be held liable outside of their own city for the debts of their fellow citizens. Cities and marketplaces throughout Europe, particularly in Germany and Italy (but also in Flanders and France) adopted stringent rules and court practices on how to deal with bad debts by foreigners. The typical sanction was to take reprisals against the fellow citizens of a defaulting foreigner, be it through seizure of any goods or, if need be, outright imprisonment. The legal rules governing communal responsibility and the rights of reprisal were studied intensively by legal historians from the Historical School, see e.g. Wach (1868), Kisch (1914), Planitz (1919).

Medieval sources mention communal responsibility among merchants in the 12th century, usually as an obstacle to merchant activity, and a bad practice that had to be abdicated or at least regulated Planitz (1919, p.97, 171). Cases of wild seizure and violence were apparently frequent. Merchants would soon avoid places that were notorious for such wild reprisals. City charters regulated and restricted the application of communal responsibility, although the practice was not abdicated altogether, see Wach (1868). Mutual privileges aimed to provide safe passage for merchants, e.g. between Flanders and places in Germany in the late 12th century Planitz (1919, p. 168). Similar agreements limiting reprisals existed between Lorraine and the Low Countries, and in numerous city charters and treaties among cities along the Rhine.

Legal historians concluded from examining hundreds of city charters that communal responsibility was ubiquitous on the European continent, and that its legal regulation was practically identical everywhere. (Kisch (1914), Planitz (1919)). Typically, evidence on the justification of the debt had to be procured, as well as of a failed previous attempt to enforce payment in the debtor's city court. Imperial law in Germany (whose enforceability was, however, notoriously low) included this clause as early as 1231, and city statutes everywhere repeated it. In Italy, cities maintained public blacklists of individual defaulters and of cities having failed to protect creditor rights.

The typical procedure included some amount of debtor rights. Although regulations often involved a right for the creditor to act on the spot before calling the judge, the rule was that the case had to be decided by the judge before dawn, otherwise the arrested merchant would be released Kisch (1914, p. 40). During major fairs and markets, judges would be walking the streets day and night to provide on-the-spot rulings, and all other court activity would be suspended while the fairs lasted.

Indeed, the major fairs enjoyed privileges of free access. Conditions on the Champagne fairs were special because of repeated interventions by the king (see Thomas (1977)). Visitors to the less important Frankfurt fairs were exempt from communal responsibility for all debts owed elsewhere in the Holy Roman Empire (but of course not in Frankfurt itself), see Brübach (1994) with examples. This gave debts owed in Frankfurt senior rank over other debts, and certainly contributed to the privileged status of the Frankfurt fairs within the payment system evolving in Germany since the 14th century. Merchants drawing a bill of exchange on the place of a major fair thus delegated enforcement to the city authorities of that fair. With its large network of trade and payments relations, its capabilities of enforcement were superior to anyone else's. We also see that disputes about bills payable in Frankfurt were settled, not in Frankfurt but in the debtors' courts, as in the examples of Basel and Constance. This is in line with the typical way communal responsibility was regulated, as Frankfurt courts would only take a case that had not been settled there previously.

These networks were magnified by the indirect links that the smaller places provided. The trade partners of 14th and 15th century Frankfurt for which Rothmann (1998) studied the records in detail include, inter alia, Constance, Basle, Cologne and Antwerp. In each of these cities, the town records include numerous payment agreements between locals and foreign merchants, which were due in Frankfurt. The network extends quickly: other clearing places mentioned in Constance included Geneva, Venice, Ravensburg, Zurzach, Zurich and many more. The books of Basle mention Lyon, Strassbourg, Brugge, Mechelen and Antwerp as clearing places besides Frankfurt. The contract partners of Basle's merchants were from Berne, Cologne, Mecheln, Strassbourg, Augsburg, Ravensburg and again from Frankfurt. The foreign partners of merchants from Constance were from Strassbourg, Ulm, St. Gallen, Ravensburg, Nuremberg, Basle, and, again, Frankfurt. The liabilities were either trade credits or bill of exchanges. In such a multilateral network, towns cared about the enforcement of contracts, independently of where the trade took place and independently of whether the citizen was a creditor or a debtor.

Communal responsibility as a backstop when trade treaties and payment arrangements failed was surprisingly long-lived. In the following, we provide micro evidence of the town of Linz in Upper Austria during the early modern time. The fairs of Linz document the ongoing use of the laws of reprisal during the 17th century (on the following see Rausch (1969)). In the 16th century, Linz, situated in the heartlands of Habsburg's Austrian possessions, had become an important fair and financial market. The fair had its high time during the 17th century, when it was a clearing place for Central and Eastern Europe. During this period, the statutes regulating the use of reprisals were renewed and confirmed by the Austrian emperor several times. The arguments put forward by the town officials in favor of the reprisal option are illuminating. They argued that reprisals were absolutely necessary to guarantee that merchants pay their liabilities. As the application of the existing rules had become lax, debtors' willingness to pay had declined. The right to apply reprisal laws would guarantee the competitiveness of the Linz fairs in Europe. Support for the legislation came also from the merchants themselves. In 1649, the visitors to the fairs set up and signed a petition to the Emperor in support of the proposed measure. The list included traders from Augsburg, Nuremberg, Cologne, Vienna, Geneva, Salzburg and many other places. Apparently, communal responsibility supported an outcome that was more preferable to merchants than any feasible alternative.

3 Money and credit demand in a village economy with search

3.1 The basic setup

We implement communal responsibility in a matching model of the demand for money and credit, adapted from Lagos and Wright (2005). In this setup, agents periodically have access to two different kinds of markets, one decentralized where agents are matched randomly, another centralized and Walrasian. As in Faig (2004) and Faig and Jerez (2005), we assume a village economy. In a decentralized market, agents from different locations trade village-specific goods with each other under random matching. Throughout, we assume that while individuals are anonymous, their citizenships are verifiable (indeed there existed medieval institutions to make that sure). In this inter-village market, agents may employ both money and/or credit as a medium of payment. We are interested in characterizing equilibria under various assumptions on the enforceability of such payments, and want to seek conditions under which the use of credit and the coexistence of money and credit are supported. The second, centralized market in this economy is an intra-village market. In this Walrasian market, only the inhabitants of each village participate, and all claims are enforceable without cost.

Let there be a finite, countable number S of villages, each inhabited by a continuum of agents on the unit interval, such that total population size is again S. Agents in each village s produce quantity q^s of a perishable village-specific good under convex disutility $N(q^s), N' > 0, N'' \geq 0$. This good is not consumed in the village where it is produced, but is a consumer good in all other villages. Also, it is a perfect substitute for the village-specific goods produced in all other villages. Agents derive utility $u(q)$ from consuming quantity q of village-specific goods produced everywhere but at home, where u is twice differentiable and satisfies the Inada conditions. This setup gives rise to complete specialization in trade.

Agents in each village also produce and consume a general, non-tradable perishable good, which is traded on a Walrasian centralized market within the respective village and is perishable as well. Agents have utility $U(C)$ over the consumption C of this non-tradable general good, with the same properties as above and $U'(C^*) = 1$ for some finite $C^* > 0$. There is disutility from producing these general goods, which is assumed to be linear in their output Y. Without loss in generality, this linear disutility is assumed equal to Y, which implies that the output of the general good is measured in (dis-)utils. This linearity assumption, introduced and discussed in detail in Lagos and Wright (2005), is crucial for the tractability of the problem, as it precludes wealth effects and ensures closed form solutions.

The temporal order of each market day is that the decentralized inter-village market for village specific goods is cleared first and the centralized intra-village market for general goods afterwards. The decentralized market is characterized by random matching between sellers and buyers from different villages. Every morning, agents become buyers or sellers with equal probability. Sellers await the arrival of foreign buyers, and produce the tradable good once a match has been established. Domestic buyers go out to some other village to match with that village's sellers there. The probability of becoming a seller and successfully being matched with a foreign buyer is ϕ. The probability of becoming a buyer and being matched successfully with a foreign seller is $1-\phi$. In the sequel, we set ϕ to one half. [1]

Not all successful matches are good ones. Conditional on a buyer/seller match establishing itself, let α denote the probability that match quality is high, as in Faig and Jerez (2004). Associated with this positive preference shock is utility $\overline{u}(q)$. In contrast, a bad match occurs with probability $1-\alpha$. It leads to lower utility $\underline{u}(q)$ and, for given FOCs, to lower consumption $\underline{q} < \overline{q}$. A buyer's expenditure in the morning market is $\overline{d}$ in a good match and $\underline{d}$ in a bad match.

The matching setup in this village economy becomes more complex once money and a financial sector are introduced. We assume two assets, money m and one-period interest-bearing commercial bills b, or outside and inside money. As all goods in the economy are perishable, these two assets are the only stores of value. Agents can create commercial bills with their signature, just like in everyday life when employing a checkbook. As money has no intrinsic value, agents would prefer to refrain from its use and resort to either barter or credit for their payments.[2] On the other hand, the use of bills may carry a resource cost, which we assume to be constant at T, and which is lost to the economy.

Let $V(m, d, \alpha, \phi)$ be the value function of an agent entering the inter-village morning market with cash and bond holdings $a = m + b$. Similarly, let $W(a, \alpha, \phi)$ be the value function of an agent entering the intra-village afternoon market, and let ρ be the intra-day discount rate.

3.2 A creditless economy, all inter-city payments in cash

In a cash economy where bills may exist as a store of value but cannot be used as a means of payment in the morning market, the agent's problem then is:

$$
\begin{aligned}
V^m(a, d, \alpha, \phi,) = \max E\{\alpha\phi\,[\overline{u}(q) \quad &+ \rho W(a - \overline{d} + i(a - \overline{m}))] \\
+(1-\alpha)\phi\,[\underline{u}(q) \quad &+ \rho W(a - \underline{d} + i(a - \overline{m}))] \\
+\alpha(1-\phi)\,[-N(q) \quad &+ \rho W(a + \overline{d} + i(a - \overline{m}))] \\
+(1-\alpha)(1-\phi)\,[-N(q) \quad &+ \rho W(a + \underline{d} + i(a - \overline{m}))]\}
\end{aligned}
\tag{1}
$$

[1] This excludes incomplete participation, unsuccessful matches, and double coincidence of wants where agents in a match are buyers and sellers at the same time. Nothing much depends on this.

[2] Notice that this does not rule out specie (i.e., gold or silver) as monetary media. Adding utility of holding specie money would complicate the notation but not fundamentally alter the results.

In this economy, all agents frontload cash to the amount $\overline{m} = \overline{d}$ payable in a good match. Any initial wealth not frontloaded for cash payments will be held as bills $b = a - \overline{m}$, and will bear interest $i(a - \overline{m})$. Agents who wake up as buyers and meet a bad match (which they do with probability $(1 - \alpha) \cdot \phi$) will emerge from the morning market with $\overline{m} - \underline{d}$ of unused cash left that bears no interest. Cash needed to meet payment needs in a bad match, $\underline{m} = \underline{d}$ can be seen as transactions demand for money. The additional $\overline{m} - \underline{m}$ needed for payment needs in a good match may be interpreted as precautionary money demand.

Agents who wake up as sellers and encounter a good match receive $\overline{d}$ in cash payments, which adds to their existing cash holdings $\overline{m}$. For sellers in a bad match, the cash receipts are $\underline{d}$.

In this setup, agents may benefit from a payment technology that reduces the initial cash holdings. First, the availability of credit in the morning market may affect the bargaining outcome between buyers and sellers, removing or limiting holdup. Second, precautionary money demand may be substituted with credit. Third, even transactions demand might be driven out, resulting in a cashless economy.

3.3 A cashless economy, all inter-city payments in credit

If credit is available at very low or zero cost, cash demand may disappear entirely. In a cashless economy (if supported in equilibrium), buyers do not frontload cash for transactions in the morning market. As cash holdings carry no interest, they also shun cash as a store of value. The agent's decision problem then becomes:

$$\begin{aligned} V^b(a, \alpha, \phi,) = \max E\{\alpha\phi\,[\overline{u}(q) &\quad + \rho W(a - \overline{d} - T + i(a - \overline{b}))] \\ +(1-\alpha)\phi\,[\underline{u}(q) &\quad + \rho W(a - \underline{d} - T + i(a - \underline{b}))] \\ +\alpha(1-\phi)\,[-N(q) &\quad + \rho W(a + \overline{d} + i(a + \overline{d}))] \\ +(1-\alpha)(1-\phi)\,[-N(q) &\quad + \rho W(a + \underline{d} + i(a + \underline{d}))]\} \end{aligned} \tag{2}$$

Buyers use bills for payment in both types of match, writing bills to the amount of $\overline{d} + T$ in a good match and $\underline{d} + T$ in a bad match. Sellers receive bills to the amount of $\overline{d}$ in a good match and $\underline{d}$ in a bad match. All assets carried over into the afternoon market are held in interest-bearing bills, with payments adding to sellers' initial holdings and reducing buyers' initial holdings. The resource cost T is a wedge between both.

3.4 A cash and credit economy

Consider an economy where cash is used for transactions in a bad match but supplemented with bills in a good match. The above problem then becomes:

$$
\begin{aligned}
V^{mb}(a,\alpha,\phi,) = \max E\{\alpha\phi\,[\overline{u}(q) &\quad + \rho W(a-\overline{d}-T+i(a-\underline{m}-\overline{b}))] \\
+(1-\alpha)\phi\,[\underline{u}(q) &\quad + \rho W(a-\underline{d}+i(a-\underline{m}))] \\
+\alpha(1-\phi)\,[-N(q) &\quad + \rho W(a+\overline{d}+i(a-\underline{m}+\overline{b}-T))] \\
+(1-\alpha)(1-\phi)\,[-N(q) &\quad + \rho W(a+\underline{d}+i(a-\underline{m}))]\}
\end{aligned}
\tag{3}
$$

In this economy, cash and credit are used alongside each other in a good match to top up cash holdings. Buyers who use credit incur resource cost T, so that in a good match, payment by bills is $\overline{b} = \overline{d} - \underline{m} + T$. Sellers in a good match receive payment $\overline{d} = \underline{m} + \overline{b} - T$. Of this, $\overline{d} - \underline{m}$ adds to their holdings of bills and earns interest. Transactions in a bad match are in cash only.

The novelty of our approach with regard to the Lagos and Wright (2005) framework lies in the assumptions about enforceability of deferred payments. In their setup, such enforcement across villages is absent altogether. Anonymous random matching precludes individual reputation effects that might support the use of credit. Agents in cross-village trade would therefore be reduced to using money. The setup then generates transactions demand for money to the level associated with the low realization of the preference shock, and additional precautionary money demand to cover the needs from the highest expected preference shock. In our setup, agents are instead assumed to be tagged with their village identity, as in Greif (2002, 2006). That is, although individual agents are anonymous, their village of origin is known.[3] This informational assumption opens room for an enforcement technology, i.e. communal responsibility or village-specific reprisals. Such reprisals can be imposed unilaterally. As will become clear further below, the presence of village-specific reprisals leads to a welfare improvement.

The afternoon home market in general goods opens after all sellers and buyers return from inter-village trade, and after interest on bond holdings is due. These holdings have decreased for agents who used bills as means of payment in the morning market. For sellers who accepted bills in the morning market, the holdings increased accordingly. We present the problem for the mixed cash-and-credit economy of eq. (3) above; results for the pure cash and pure credit economies are analogous.

[3] Reputation, trigger-strategy etc. equilibria may exist across villages if city authorities themselves are allowed to behave strategically. We mention the possibility of such equilibria merely as a reference case and focus on the non-cooperative outcome instead.

Buyer's problem in afternoon market: From eq. (3), the remaining endowment of a buyer who enters the centralized Walrasian afternoon market is:

$$a - \overline{d} - T + i(a - \underline{m} - \overline{b}) = (1+i)(a - \overline{d} - T) \ ,$$

if match quality in the morning market was high and

$$a - \underline{d} + i(a - \underline{m}) = (1+i)(a - \underline{d}) \ ,$$

if match quality in the morning was low and credit was not used. Here, we have employed the identity $\overline{d} = \underline{m} + \overline{b}$ and the fact that agents will not hold more money than necessary, $m - \bar{m} = 0$.

The value of entering the afternoon market then is:

$$W[(1+i)(a - \overline{d} - T)] = \max_{C,Y,a'}[U(C) - Y + \beta V(a', \alpha, \phi)]$$

subject to:

$$C + a' = Y + (1+i)(a - \overline{d} - T)$$

where C is consumption and Y is production of the general good, and where a' is next morning's starting endowment. Conditions are analogous with $d = \underline{d}$ and $T = 0$ for a buyer in a bad match who paid in cash.

Rearranging terms, we obtain:

$$\begin{aligned} W[(1+i)(a - \overline{d} - T)] = & \max_{C,a'}[U(C) - C - a' + \beta V(a', \alpha, \phi)] \\ & + (1+i)(a - \overline{d} - T) \end{aligned} \tag{4}$$

with the FONC:

$$\begin{aligned} U'(C*) &= 1 \\ \beta V'(a') &= 1 \end{aligned}$$

Marginal utility and hence consumption of the general good C is the same for all agents, and is invariant to the endowment brought into the afternoon market. The marginal utility of carrying a unit of endowment over into the next period is equal to one, and is again independent of the endowment brought into the afternoon market. This means that although agents were heterogeneous when entering the afternoon market, they will be homogeneous again when entering next period's morning market. Whoever brought "too much" endowment from the morning market trades it in and consumes the excess, while agents bringing "too little" endowment from the morning market produce more to arrive at the desired consumption level.

Quasilinear preferences in this Lagos/Wright (2005) afternoon market ensure that maximization of $V(a')$ with respect to a' is independent of a, thus excluding wealth effects. In addition, $W(a)$ is linear in a, and we obtain linear envelope conditions:

$$W'(a) = -W'(d) = 1 + i$$

Seller's problem in afternoon market: In the setup of eq. (3), a seller emerges from a good match in the inter-village morning market and enters the centralized afternoon market with endowment

$$a + \overline{d} + i(a - \underline{m} + \overline{b}) = (1+i)(a + \bar{d}) - 2\, i\, \underline{m} \ ,$$

Similarly, a seller emerges from a bad match with endowment

$$a + \underline{d} + i(a - \underline{m}) = (1+i)(a + \underline{d}) - 2\, i\, \underline{m} \ ,$$

The value of entering the afternoon market then is:

$$W[(1+i)(a + \overline{d}) - 2\, i\, \underline{m}] = \max_{C,Y,a'} [U(C) - C - a' + \beta V(a', \alpha, \phi)]$$

subject to:

$$C + a' = Y + (1+i)(a + \overline{d}) - 2\, i\, \underline{m}$$

Rearranging terms, we obtain:

$$\begin{aligned} W[(1+i)(a + \overline{d}) - 2\, i\, \underline{m}] = & \max_{C,a'} [U(C) - C - a' + \beta V(a', \alpha, \phi)] \\ & + (1+i)(a + \bar{d}) - 2\, i\, \underline{m} \end{aligned} \tag{5}$$

Again, the FONC are:

$$\begin{aligned} U'(C^*) &= 1 \\ \beta V'(a') &= 1 \end{aligned}$$

Agents' problem in morning market: To solve the agents' problem in the anonymous matching market, we follow the Nash bargaining setup of Lagos and Wright (2005), where θ is buyer i's bargaining power. The threat points are given by the continuation values in the afternoon market, $W(.)$ that would apply in the case of no match at all. There are two Nash bargains to be considered, one in the case of a good match, the other in the case of a bad match, depending on the buyer's preference shock. In addition, a we consider a candidate solution where credit is used in the good state. We denote this by a qualitative variable Ψ with

$$\Psi = \begin{cases} 1 & \text{if } \tilde{b} > 0 \text{ and } T > 0 \\ 0 & \text{else} \end{cases}$$

$\tilde{d}$ takes the values $\overline{d}$ in a good match and $\underline{d}$ in a bad match. We adopt a similar notation for all other state-dependent variables.

From eq. (3) above, the seller-buyer bargaining problem in the morning market (where credit is used in the good state) then is:

$$B^{mb} = \Big[\, \tilde{u}(\tilde{q}) + \rho W((1+i)(a - \tilde{d} - \Psi T)) - \rho W((1+i)a - i\,\underline{m})\Big]^{\theta}$$
$$\times \Big[-N(\tilde{q}) + \rho W((1+i)(a+\tilde{d}) - 2\,i\,\underline{m}) - \rho W((1+i)a - i\,\underline{m})\Big]^{1-\theta}$$

Thanks to the linearity of the continuation values, solving the bargaining problem simplifies to:

$$\max_{q,d} \Big[\tilde{u}(\tilde{q}) - \rho((1+i)(\tilde{d} + \Psi T) + i\,\underline{m})\Big]^{\theta} \times \Big[-N(\tilde{q}) + \rho((1+i)\tilde{d} - i\,\underline{m})\Big]^{1-\theta} \tag{6}$$

subject to:

$$\tilde{d} \leq \underline{m} + \Psi(\overline{b} - T)$$

The constraint means that payment to the seller is in cash in the bad match and in cash-cum-bills in the good match, indicated by $\Psi > 0$. T is the resource cost of writing bills that is lost to the economy.

Lemma 1 *The optimality conditions of the above bargaining problem are:*

$$\frac{u'(\tilde{q}^*)}{N'(\tilde{q})^*} = 1 + \frac{1-\Psi}{1+\Psi i}\frac{\lambda}{\theta}\left[\frac{u\tilde{q} - \tilde{d} - \Psi(T + i\tilde{b})}{-N\tilde{q} + \tilde{d} + \Psi i\tilde{b}}\right]$$

Proof See Appendix.

Lemma 1 states that there is a wedge between marginal utility of consumption and marginal cost of production whenever the cash constraint is binding. There is no such wedge in the case of take-it-or-leave-it offers, i.e. if $\theta \in \{0, 1\}$.[4] Then, market outcomes are Pareto efficient, as there is no money holdup. There is also no wedge if $\Psi = 1$ i.e. if the transaction is wholly or partly financed by credit. What distinguishes Lemma 1 from the standard result is that the payments regime Ψ is a choice variable, whose value buyers can determine *after* their preference shock is revealed. Whether or not the cash constraint becomes binding is thus subject to the buyer's decision on writing a bill of exchange or not. This property also distinguishes the present setup from the cash/credit equilibrium of Berentsen, Camera and Waller (2007), where credit can be obtained beforehand but a cash constraint still exists

[4]Remarkably, pre-modern markets had brokerage institutions that made sure of this, mostly by allocating the surplus to buyers.

once buyers enter the market. We do not have such a constraint here. However, depending on the cost of credit, we still may have money holdup problems. As will become clear below, it may sometimes indeed be preferable for a buyer to accept the holdup and not use credit.

4 The demand for money and credit

Before finding equilibria, it remains to be determined under which conditions payment with cash or with bills of exchange could occur. For each agent, the match qualities define a Markov chain of transaction volumes $\tilde{d} \in \{\underline{d}, \overline{d}\}$, and in each state, the cheaper medium of payment will be preferred.

4.1 A creditless economy, all inter-city payments in cash

We first return to the case where bills are available as a store of value but no credit can be used the morning market, $\Psi = \tilde{b} = 0$. Then, buyers need to frontload sufficient cash to cover payment needs under the positive preference shock. From eq. (1) above, the bargaining problem is obtained as:

$$B^m = \left[\tilde{u}(\tilde{q}) + \rho W(a - \tilde{d} + i(a - \overline{m})) - \rho W(a + i(a - \overline{m}))\right]^{\theta} \\ \times \left[-N(\tilde{q}) + \rho W(a + \overline{d} + i(a - \overline{m})) - \rho W(a + i(a - \overline{m}))\right]^{1-\theta}$$

Due to the linearity of $W(.)$, the solution to this problem becomes:

$$\max_{q,d} \left[\tilde{u}(\tilde{q}) - \rho\tilde{d}\right]^{\theta} \times \left[-N(\tilde{q}) + \rho\tilde{d}\right]^{1-\theta} \tag{7}$$

subject to:

$$\tilde{d} \leq \tilde{m}$$

In this form, the bargaining problem is as in Lagos and Wright (2005), except for the intraday discounting factor ρ. The cash constraint will bind if in a good match, as $\overline{d} = \overline{m}$. As agents will always take enough money to the market in order to provide for this contingency, the cash constraint will never bind in a bad match. From Lemma 1 we obtain the optimality conditions as:

$$\frac{\underline{u}'(\underline{q})}{N'(\underline{q})} \stackrel{!}{=} 1\,; \qquad \frac{\overline{u}'(\overline{q})}{N'(\overline{q})} \stackrel{!}{=} 1 + \frac{\lambda}{\theta}\left[\frac{\overline{u} - \overline{m}}{-N\overline{q} + \overline{m}}\right]^{1-\theta}$$

4.2 A cashless economy, all inter-city payments in credit

This economy is the other extreme case in which the use of credit drives cash out of circulation entirely. If supported in equilibrium, buyers do not frontload cash before entering the morning market, and also shy away from cash as a store of value. From eq. (2) above, the bargaining problem then becomes:

$$B^b = \Big[\tilde{u}(\tilde{q}) + \rho W((1+i)(a - \tilde{d} - \Psi T)) - \rho W((1+i)a)\Big]^\theta \\ \times \left[-N(\tilde{q}) + \rho W((1+i)(a+\overline{d})) - \rho W((1+i)a)\right]^{1-\theta}$$

which, thanks to the linearity of $W(.)$, becomes:

$$\max_q \left[\tilde{u}(\tilde{q}) - \rho((1+i)\,\tilde{b} + T)\right]^\theta \times \left[-N(\tilde{q}) + \rho(1+i)\tilde{b}\right]^{1-\theta} \tag{8}$$

From Lemma 1 we obtain the FOC as:

$$\frac{\underline{u}'(\underline{q})}{N'(\underline{q})} \stackrel{!}{=} \frac{\overline{u}'(\overline{q})}{N'(\overline{q})} \stackrel{!}{=} 1$$

4.3 Coexistence of cash and credit: necessary conditions

Cash and credit may coexist in inter-city payments under two assumptions about cost (if such an equilibrium exists, which is yet to be determined). Using bills of payment reduces the interest cost associated with of having to provide cash upfront. Also, agents avoid some or all of the holdups in the above bargaining problems. Agents encounter bargaining holdup under a good match in a pure cash economy as in eq. (1), no holdup at all in a pure credit economy as in eq. (2), and a bargaining holdup under a bad match in the mixed cash and credit economy (3). Which setup they prefer depends on the gain in consumer surplus from switching to the use of credit. If the resource cost T of credit is very low, cash might be driven out of circulation entirely. Consider a buyer who takes just sufficient cash to the market to pay for a bad match, and reserves writing a bill of exchange for a good match. Then, if a bad match is realized, the resulting bargaining will be subject to holdup, characterized by the outcome $\underline{q}$. Given the availability of credit, the buyer always has the option of not taking any cash to market and thus avoiding the holdup. This comes at cost T but leads to higher transactions volume $\underline{u}^*$. The buyer will refrain from using credit and accept the money holdup if the cost of breaking out of the holdup exceeds the increase in surplus from doing so:

Assumption 1 : $(1-\alpha)\rho(1+i)T > [\underline{u}(\underline{q}^*) - \underline{u}(\underline{q})]$

The LHS of this expression is the expected resource cost of using credit. The RHS is the utility loss of accepting the holdup. In an economy with take-it-or-leave-it offers (or market institutions that mimic them), the inequality collapses into a positivity constraint on T.

Conversely, if using credit instruments is very expensive, agents may accept the bargaining holdups and interest cost of holding cash instead. Credit is used as long as:

Assumption 2 *:* $\alpha\rho(1+i)T < (1-\alpha)\rho i[\overline{m} - \underline{m}] + [\overline{u}(\overline{q}^*) - \overline{u}(\overline{q})] - [\underline{u}(\underline{q}^*) - \underline{u}(\underline{q})]$

The left-hand side of this condition is the expected resource cost of credit in a good match. The first two terms on the RHS evaluates utility from using credit versus cash in the two states, while the last term is the interest cost of precautionary cash holdings left unused in a low-quality match.

Proposition 1 *: Under Assumptions 2 and 1, optimal money demand (if supported in equilibrium) is characterized by the following properties:*
(a) Money demand is positive at $\underline{m} = \underline{q}$*, irrespective of the realization of* $\tilde{q}$*.*
(b) Gross demand for bills of exchange equals $\alpha\tilde{b} = \alpha[\overline{d} - \underline{d}] = \alpha[\overline{m} - \underline{m}]$*.*

Proof: see Appendix.

Consequently, the bargaining problem in the morning market now is as in eq. (1), with the FOC given by:

$$\frac{\overline{u}'(\overline{q})}{N'(\overline{q})} \stackrel{!}{=} 1\,; \qquad \frac{\underline{u}'(\underline{q})}{N'(\underline{q})} \stackrel{!}{=} 1 + \frac{\lambda}{\theta}\left[\frac{\underline{u} - \underline{m}}{-N\underline{q} + \underline{m}}\right]^{1-\theta}$$

Armed with these results, it is now straightforward to obtain the intra-day Euler equation as:[5]

$$-u'(\bar{q}^*) + \rho(1+i)U'(C^*) = 0$$

Due to the quasilinearity of the general good, this simplifies to:

$$u'(\bar{q}^*) = \rho(1+i)$$

As buyers in a bad match do not participate in the bond market, and as $u'(\bar{q}^*)$ is given by the Nash bargaining outcome, this pins down the intra-day interest rate i.

[5]We keep in mind that due to quasilinearity in the afternoon market, $V(a')$ is independent of a.

5 Equilibrium

5.1 A restricted social planner problem

We proceed gradually towards characterizing equilibrium in this city economy. The first step is to study a social planner's solution. We limit attention to take-it-or-leave-it offers where bargaining power is restricted to either zero or one, $\theta \in \{0, 1\}$. This rules out holdups that would reduce output below the socially optimal level. Let $\lambda^{ij} di$ be the social planner's weight of agent idi in city j.[6] Furthermore, let Assumptions 1 and 2 hold. Then, the social planner problem is:

$$\begin{aligned}\lambda^{ij} di V_i j(a) = EV^{mb}(a, \alpha, \phi,) = \max_{q,d} E\{\alpha\phi\,[\overline{u}(q) &\quad + \rho W(a - \overline{d} - T + i(a - \underline{m} - \overline{b}))] \\ +(1-\alpha)\phi\,[\underline{u}(q) &\quad + \rho W(a - \underline{d} + i(a - \underline{m}))] \\ +\alpha(1-\phi)\,[-N(q) &\quad + \rho W(a + \overline{d} + i(a - \underline{m} + \overline{b}))] \\ +(1-\alpha)(1-\phi)\,[-N(q) &\quad + \rho W(a + \underline{d} + i(a - \underline{m}))]\}\end{aligned} \tag{9}$$

If an equilibrium exists, buyers' money and credit demand is given by Proposition 1. Given the continuity of agents, a law of large numbers applies, and aggregate demand for outside money is:

$$\int_0^1 \tilde{m}\, i\, di = \alpha \int_0^1 \underline{q}\, i\, di + (1-\alpha) \int_0^1 \underline{q}\, i\, di = \underline{q}$$

By ordering the population accordingly, we can write:

$$\int_0^1 \tilde{m}\, i\, di = \int_0^\alpha \underline{q}\, i\, di + \int_\alpha^1 \underline{q} i di = \underline{q}$$

Demand and supply of bills of exchange for transactions demand is:

$$\tilde{b}_{aggr} = \alpha(\overline{q} - \underline{q}) - \alpha(\overline{q} - \underline{q}) = 0$$

i.e. bills of exchange (or inside money, which is what they are) are in zero net supply. The gross supply of bills of exchange for transactions is equal to the proportion of good matches in the population times the excess output under good conditions, i.e.:

$$\tilde{b}_{gross} = \alpha(\overline{q} - \underline{q})$$

Together, the demand for outside and inside money is:

$$\underline{m} + \tilde{b}_{gross} = \underline{q} + \alpha\overline{q}$$

[6] Agents are continuous over the unit interval, and we assume $\int_0 \lambda^{ij} di = 1$.

which equals total output in the economy. Notice that in this setup, outside money is linked to transactions demand, while credit or inside money is used for precautionary money demand.

5.2 Decentralized market solution, no collective responsibility

Next we characterize equilibria in a decentralized economy, assuming for the moment that no collective responsibility exists, and hence no reprisals take place if debtors fail to honor their bills of exchange. As there is a continuum or anonymous individual agents, individual reputation effects or other strategies that could support a cooperative equilibrium are ruled out: the inter-city payment system collapses into a pure cash economy. This holds true if we assume that city governments cannot interact strategically, just as individuals in this setup cannot. As there is a countable, finite number of cities N, we cannot rule out strategic interactions like coalition formation, reputation, tit for tat strategies, and the like. To the extent that city governments behave strategically and engage in such behavior, the folk theorem points to infinitely many equilibria that may establish themselves, and that dominate the non-cooperative solution sketched here in terms of welfare. Indeed, European history offers rich accounts of city leagues that alternately privileged and fought each other, and that had fluctuating memberships. If N is high we would indeed expect such equilibria to be rather unstable. While we fully acknowledge the possible relevance of such equilibria, they are not our primary focus of interest. What we want to highlight here is the possibility of mimicking a fully cooperative outcome in the context of non-cooperative individual behavior.

An agent trading out of town in a good match faces the following buyer's problem under no enforcement:

$$V_b^{NE} = u(\overline{q}) + \begin{cases} \rho W[(1+i)(a-\overline{b}-\underline{m})] & \text{if redeems the bill} \\ \rho W[(1+i)(a-\underline{m})] & \text{if defaults} \end{cases}$$

which gives the buyer an incentive to pay with credit and then default. The buyer's moral hazard induces the seller not to accept payment by credit-cum-cash $\overline{b}+\underline{m}$ and insist on cash payment $\overline{m}$ instead.

5.3 Collective responsibility as an enforcement device

We now introduce collective responsibility into this framework. As mentioned earlier, collective responsibility means that all agents of a city can be held liable for one another in inter-city trade. This implies that if any bill of payment has been defaulted on, the creditor's city immediately sanctions this by seizing the resources of some other representatives of this city, and preventing these agents' market participation. Let b^d be a defaulted bond presented for payment to an agent while trading abroad. In our setup, an agent out of town is always a buyer. As the defaulted sum is $b^d = \tilde{b} = \overline{m} - \underline{m} \neq \underline{m}$, it may take more than one act of reprisal for the creditor city to fully recover the defaulted credit. In each of these acts, the loss to the victim of reprisal equals the foregone utility from consumption of the village specific good, plus the seized cash portion $z \cdot \underline{m}$, $0 < z \leq 1$. The value function of a victim of reprisal $V_b^{NE,\text{reprisal}}$ then is:

$$V_b^{NE,\text{reprisal}} = u(0)+\rho W((1+i)(a-\underline{m})+(1-z)\underline{m}) \text{ , no enforcement in afternoon markets}$$

By contrast, under full enforcement in local afternoon markets the victim of reprisal is able to obtain full compensation from the agent who defaulted in the first place. The victim's value function under full enforcement at home, $V_b^{FE,\text{reprisal}}$, then is:

$$V_b^{FE,\text{reprisal}} = V_b^{MB} \text{ , full enforcement in afternoon markets}$$

where V_b^{MB} is the buyer's value function in an economy with mixed use of money and credit as in the upper two lines of eq. (3) above. Full compensation includes the consumer surplus forgone in the morning market, plus monetary compensation for the funds sequestered abroad. Evidently, this gives any agent who is presented with a defaulted bill of a fellow citizen the incentive to recover the payment after returning home. In other words, the agent becomes the *enforcement agent* of the host city upon returning home.

We have already seen that under collective responsibility, an agent held liable for the default of his fellow citizen has an incentive to recover his loss from the defaulter. As the defaulted sum is $\tilde{b} = \overline{m} - \underline{m} \neq \underline{m}$, it may take more than one act of reprisal for the creditor city to fully recover the defaulted credit. In each of this act, the loss to the victim of reprisal equals the expected consumer surplus from the transaction, plus the seized cash $s \times s \cdot \underline{m}$:

$$\begin{aligned} V_b^{FE,\text{reprisal}} - V_b^{NE,\text{reprisal}} =& \alpha u(\overline{q}) + (1-\alpha)u(\underline{q}) - u(0) \\ &+ \rho W[(1+i)(a-\underline{m}) - \alpha(1+i)\overline{b}] \\ &- \rho W[(1+i)(a-\underline{m}) + (1-z)\underline{m}] > z \cdot \underline{m} \end{aligned}$$

Whenever institutions are such that consumer surplus is positive, the individual welfare loss from reprisal will exceed the cash amount $z \cdot \underline{m}$ seized. to see this, note that:

$$\begin{aligned} \rho[W[(1+i)(a-\underline{m}) - \alpha(1+i)\overline{b}] - &\rho W[(1+i)(a-\underline{m}) + (1-z)\underline{m}] \\ &= (1-z)\underline{m} \end{aligned}$$

Thus, under full enforceability in the home market, all agents held liable abroad will reclaim their losses from the defaulter in the afternoon market. In comparison, the gain from defaulting for an agent if caught is:

$$\overline{b} = \underbrace{\overline{m} - \underline{m}}_{\text{Agent's gain if not caught}} < \underbrace{\left([u(\overline{q}) - (\tilde{b} + \underline{m})] + [u(\underline{q}) - \underline{m}]\right)}_{\text{Agent's loss if caught}}$$

The LHS of this inequality is the agent's gain from defaulting on a bill of exchange signed when shopping out of town. The RHS of the expression is the same amount again plus the compensation payable to the victims of reprisal, which equals the deadweight loss from their being excluded from the outside market for that period. As can be seen, the net gain from defaulting is negative. Hence, under collective reprisal and collective responsibility, there is no incentive to default on bills of exchange, so that - within the narrow confines of our setup, which excludes bankruptcy for other reasons - default does not occur in equilibrium. As a consequence, the equilibrium under reprisal fully supports the use of bills of exchange, and precautionary money demand equals:

$$m_{aggr}^{CR} = \alpha \underline{q} + (1-\alpha)\underline{q} = \underline{q}$$

while the net supply of bonds is zero:

$$\tilde{b}_{aggr}^{CR} = -\alpha(\overline{q} - \underline{q}) + (1-\alpha)(\underline{q} - \overline{q}) = 0$$

and gross demand for bonds is:

$$\tilde{b}_{gross}^{CR} = \alpha(\overline{q} - \underline{q})$$

In short, equilibrium under collective responsibility replicates the (restricted) social planner equilibrium.

5.4 Existence of Equilibrium

The above discussions show that in all possible cases, the continuation values are linear in endowments. This means the problem at hand is one of unbounded returns, so that the feasibility constraints need to be evaluated first, as in Stokey and Lucas (1989, ch.4.4), before the Contraction Mapping Theorem can be applied. To proceed, we first need to establish boundedness of money demand.

Lemma 2 *Initial money holdings (money demand)* $\tilde{m}$ *are bounded by* $0 < \tilde{m} \leq \overline{m} < \infty$.

Proof: See Appendix.

On the basis of Lemma 1, Lemma 2, and Proposition 1, we have

Proposition 2 *Let* $X \subset R$ *be the continuous and compact set of possible values for state variable a under the feasibility constraints of Lemmas 1 and 2, and consider the space of bounded continuous functions given by* $\mathbb{C}(X) = \{\nu X \mapsto \mathbb{R}\}$*, together with the sup norm* $||\nu|| = \sup_{a \in X} |\nu(a)|$.

(a) X *is nonempty,*

(b) The operator $\mathbb{T}$ *on* $\mathbb{C}(X)$ *defined by*

$$\mathbb{T}V(a) = \max_{a' \in X}\{E[\nu(a)] + a - L(a) - a' - c + \beta V(a')\}$$

where $c = \pi\Psi T$*, where* $0 < L(a) < 1$ *is a collection of linear terms in a that will be lower than unity in equilibrium, and where the value of* Ψ *is determined by Proposition 1 and Lemma 2, is a contraction mapping on* $\mathbb{C}(X)$ *with a unique fixed point* $\nu^* \in X$.

Proof: See Appendix.

Note that as expectations in eqs. (3-2) above are defined on a discrete event space, inclusion of the expectation operator in Proposition 2 is innocuous; it is merely shorthand for writing out all the cases in full.

6 Discussion

Our results critically depend on the linearity of the value functions, which excludes wealth effects and permits the considerations underlying Proposition 1. There are several established ways of achieving linearity of $W(a)$. We followed the setup of Lagos and Wright (2005), which ensures linearity of $W(a)$ by assuming quasilinear preferences for a good produced in an afternoon market. Other mechanisms have been devised in the literature, and would give rise to historically meaningful applications as well. This section briefly presents two of those alternative approaches and provides a discussion of their possible interpretations.

One paradigm is the extended family setup of Shi (1997). In our context, it would imply that the family as the basic decision making unit is pooled over sellers and buyers, so that wealth effects on the family as a whole are absent. Then, the device of a second, afternoon market becomes unnecessary. Under an extended family assumption, the basic value function maximization problem holds not just in expectation but, through the law of large numbers, as a deterministic relationship. Hence, under all realizations the continuation value of an extended family is constant.

From a historical perspective, the extended family setup is empirically highly relevant, and much more than a solution device adopted out of convenience. Communal responsibility has itself been interpreted by legal historians as an institution originating from family feuds, see e.g. Planitz (1919). Family firms, frequent since the late Middle Ages, were indeed one way to overcome incentive problems that originated from the weakening of the extended family, see Boerner and Ritschl (2009).

However, the connection between the extended family and the firm goes even further. Building on the work of legal historians on the origins on the firm, one of us, Boerner (2005), shows that the contractual relations characterizing early firms were indeed modeled on medieval family trusts, by which an indivisible family possession, such as a manor, would become the jointly owned asset of the heirs, forcing them to pool all their other assets and receiving a quasi-fixed rent in return.

Applying this logic to the problem of collective responsibility, it is obvious that both the extended family and the firm are stronger contractual relationships than mere collective responsibility towards a third party. In fact, as pointed out in Boerner (2005), collective responsibility itself is not a precursor of the firm. Instead, the firm originated to solve what in the model of the previous sections is a problem with the linearity of the agents' continuation value.

A second way to introduce linearity into the continuation values W (.) is the idea of Faig (2004) that before entering a match, each agent can insure the state-dependent outcome within their home city. In our context where match quality follows a two-state Markov chain, think of an insurance premium μ that buys actuarily fair state-dependent payoffs such that:

$$\mu = \alpha\overline{q} + 1 - \alpha\underline{q}$$

Under this insurance scheme, agents' budget constraints become state-invariant; i.e. agents have a fixed income no matter what the outcome of their inter-city trade is. Then, the afternoon market can again be dispensed with. This insurance framework is very similar to invoking an extended family - or a firm. Agents trading across cities and representing firms would in fact face such a state-invariant budget constraint. Of course, the feasibility of such a scheme would inextricably depend on the tightness of the firm's internal contractual relationships to deal with the obvious incentive problems. Again, however, the insurance assumption is more than just an analytical device adopted out of convenience. There is evidence that early modern firm contracts indeed sought to implement fixed-income schemes with low-powered incentives but full liability of any agent.

However, setting up extended families or firms (or a combination of both) in the framework of the previous sections comes at a cost. As we would have to assume the number of families or firms to be finite, strategic behavior and nontrivial dynamic equilibria are introduced. In an extended family, the family, not the individual, is the basic decision maker. Hence, strategic interactions between families will occur, be it within a city or (again assuming a finite number of cities) across cities. The same is true in the case of state-contingent insurance contracts for all agents of one city. This contract essentially makes the city collective the basic decision unit, and strategic interactions between cities should then be observed. Historical evidence for both phenomena is abundant. Our own approach to modeling inter-city trade deliberately abstracts from this, as our aim is only to show that even under the weaker assumption of collective responsibility, trade and payment with credit is supported.

7 Conclusion

Financial assets as a medium of payment in long-distance trade emerged in medieval Europe in spite of fragmented city jurisdictions. This paper studies the effects of contract enforcement through communal responsibility, a medieval institution which held all traders of a given city liable for each others' debts when trading abroad. We both presented historical evidence and modeled the effects of this institution on an anonymous matching market.

The contribution of our paper is twofold. We find that communal responsibility, by which each city's merchant was held liable abroad for any claims on his fellow citizens, was highly effective in enforcing contracts across cities. This enabled financial markets to integrate even in the absence of territorial jurisdictions. In the context of a random matching model of trade, we find that communal responsibility is a collateralizing device that supports credit in equilibrium even if individual traders are anonymous, so long as they can be identified by their citizenship.

We study the effects of communal responsibility in the matching model of money demand of Lagos and Wright (2005), interpreted as a village economy as in Faig (2004) and Faig and Jerez (2005). Communal responsibility creates a liability pool with effective enforcement, which supports coexistence of money and credit in monetary search equilibrium. Under this enforcement technology, cash and credit may coexist in equilibrium if there is a fixed but sufficiently low fixed cost of using credit.

Indeed we find that medieval cities acted as market makers, adopting city statutes that ensured the collateralization of credit through the threat of communal responsibility. Europe's emerging financial markets integrated regionally and across time, even adopting centralized clearing markets. The fragmentation of jurisdictions did little to disturb this process, as communal responsibility gave market participants sufficiently strong incentives not to default.

We see this research as a first step towards providing an integrated conceptual framework for the analysis of money and credit demand in anonymous matching markets with liability pools. Historical institutions such as communal responsibility, big families, and other risk sharing devices were replaced with contemporary technologies like checkbooks and credit cards. All of these solve the problem of credit enforcement in anonymous random matching by relegating it to a second market, in which enforcement of claims is possible.

References

Berentsen, A., G. Camera, and C. Waller (2007): "Money, Credit, and Banking," *Journal of Economic Theory*, 135 (1), 171–195.

Boerner, L., and J. Hatfield (2010): "The Economics of Debt Clearing Mechanisms," *Working Paper: Stanford/ FU-Berlin Discussion Paper 2010/27.*

Boerner, L., and A. Ritschl (2002): "Individual Enforcement of Collective Liability in Pre-Modern Europe," *Journal of Institutional and Theoretical Economics*, 158 (1), 205–213.

——— (2009): "The Economic History of Sovereignty: Communal Responsibility, the Extended Family, and the Firm," *Journal of Institutional and Theoreitical Economics*, 165 (1), 99–112.

Boyer-Xambeu, M.-T., G. Deleplace, and L. Gillard (1991): "Goldstandard, Währung und Finanz im 16. Jahrhundert," in *Kredit im spätmittelalterlichen und frühneuzeitlichen Europa*, ed. by M. North. Böhlau, Vienna.

Brübach, N. (1994): *Die Reichsmessen von Frankfurt am Main, Leipzig und Braunschweig*. Steiner, Stuttgart.

Cavalcanti, R., and N. Wallace (1999): "Inside and Outside Money as Alternative Media of Exchange," *Journal of Money, Credit and Banking*, 31, 343–357.

Chatterjee, S., and D. Corbae (1996): "Money and Finance with Costly Commitment," *Journal of Monetary Economics*, 37, 225.–248.

de Roover, R. (1953): *L'Evolution de la Lettre de Change. XIVe-XVIIIe siècles.* Librairie Armand Colin, Paris.

Faig, M. (2004): "Money and Banking in an Economy with Villages," *Manuscript.*

Faig, M., and B. Jerez (2006): "Inflation, Prices, and Information in Competitive Search," *Advances in Macroeconomics*, 6 (1).

Freeman, S., and G. W. Huffman (1991): "Inside Money, Output and Causality," *International Economic Review*, 32 (3), 645–667.

Freundt, C. (1899): *Das Wechselrecht der Postglossatoren*. Duncker & Humblot, Leipzig.

Goldschmidt, L. (1891): *Universalgeschichte des Handelsrechts*. Enke.

Greif, A. (2002): "Institutional Foundations of Impersonal Exchange. From Communal to Individual Responsibility in Pre-Modern Europe," *Journal of Institutional and Theoretical Economics*, 158 (1), 168–204.

——— (2006): "History Lessons: The Birth of Impersonal Exchange: The Community Responsibility System and Impartial Justice," *The Journal of Economic Perspectives*, 20 (2), 221–236.

He, P., L. Huang, and R. Wright (2005): "Money and Banking in Search Equilibrium," *International Economic Review*, 46 (2), 637–670.

Ireland, P. (1995): "Endogenous Financial Innovation and the Demand for Money," *Journal of Money, Credit and Banking*, 27 (1), 1241–1289.

Kisch, G. (1914): *Der deutsche Arrestprozess.* Tempsky, Vienna.

Kiyotaki, N., and R. Wright (1989): "On Money as a Medium of Exchange," *Journal of Political Economy*, 97 (4), 927–954.

——— (1991): "A Contribution to the Pure Theory of Money," *Journal of Economic Theory*, 53 (2), 215–235.

Kocherlakota, N. (1998): "Money is Memory," *Journal of Economic Theory*, 81, 232–251.

Lacker, J., and S. Schreft (1996): "Money and Credit as Means of Payment," *Journal of Monetary Economics*, 38 (1), 3–23.

Lagos, R., and G. Rocheteau (2008): "Money and Capital as Competing Media of Exchange," *Journal of Economic Theory*, 142 (1), 247–258.

Lagos, R., and R. Wright (2005): "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 113, 463–484.

Munro, J. (1991): "Die Anfänge der Übertragbarkeit: Einige Kreditinnovationen im englisch-flämischen Handel des Spätmittelalters (1360-1540)," in *Kredit im spätmittelalterlichen und frühneuzeitlichen Europa*, ed. by M. North. Böhlau, Vienna.

——— (1994): "The International Law Merchant and the Evolution of Negotiable Credit in Late-Medieval England and the Low Countries," in *Textiles, Towns, and Trade: Essays in the Economic History of Late-Medieval England and the Low Countries*, ed. by J. Munro. Ashgate, Aldershot.

North, D. (1981): *Structure and Change in Economic History.* Norton, New York.

Optiz, P. (1968): *Der Funktionswandel des Wechselindossaments.* Duncker & Humblot, Berlin.

Planitz, H. (1919): "Studien zur Geschichte des deutschen Arrestprozesses," *Zeitschrift der Savigny-Stiftung für Rechtsgeschichte*, 40, 87–198.

Prescott, E. C. (1987): "A Multiple Means of Payments Model," in *New Approaches to Monetary Economics*, ed. by W. A. Barnett, and K. J. Singleton. Cambridge University Press, New York.

RAUSCH, W. (1969): *Die Geschichte der Linzer Märkte im Mittelalter*. Wimmer, Linz.

ROTHMANN, M. (1998): *Die Frankfurter Messen im Mittelalter*. Steiner, Stuttgart.

SCHAPS, G. (1892): *Zur Geschichte des Wechselindossaments*. Enke, Stuttgart.

SCHAUBE, A. (1898): "Zur Geschichte und Natur des ältesten Cambium," *Jahrbücher fr Nationalökonomie und Statistik*, 70, 603–621.

SHI, S. (1997): "A Divisible Search Model of Fiat Money," *Econometrica*, 65 (1), 75–102.

TELYUKOVA, I., AND R. WRIGHT (2008): "A Model of Money and Credit, with Applications to the Credit Card Puzzle," *Review of Economic Studies*, 75 (2), 629–649.

USHER, A. (1943): *The Early History of Deposit Banking in Mediterranean Europe*. Harvard University Press, Cambridge MA.

VAN DER WEE, H. (1963): *The Growth of the Antwerp Market and the European Economy*. Nijhoff, The Hague.

WACH, A. (1868): *Der Arrestprozess in seiner geschichtlichen Entwicklung*. Leipzig.

Appendix

Proof of Lemma 1:

Let B be the Nash bargaining problem:

$$B = \max_{q,d} \left[\tilde{u}(\tilde{q}) - \rho((1+i)\tilde{d} + \Psi T - i\,\underline{m})\right]^{\theta}$$
$$\times \left[-N(\tilde{q}) + \rho((1+i)\tilde{d} - i\,\underline{m})\right]^{1-\theta}$$

Let $D \equiv \tilde{u}(\tilde{q}) - \rho((1+i)\tilde{d} + \Psi T - i\,\underline{m})$ and $E \equiv -N(\tilde{q}) + \rho((1+i)\tilde{d} - i\,\underline{m})$. Take logarithmic derivatives of B with respect to $\tilde{q}$ and $\tilde{d}$ to find the FOC:

$$\frac{\partial ln\nu(a,d)}{\partial \tilde{q}} = \frac{\theta \tilde{u}'(\tilde{q})}{D} - \frac{(1-\theta)N'(\tilde{q})}{E} \stackrel{!}{=} 0 \qquad \text{(I)}$$

$$\frac{\partial \ln \nu(a,d)}{\partial \tilde{d}} = -\frac{\rho\theta(1+i)}{D} + \frac{\rho(1-\theta)(1+i)}{E} - \frac{\lambda}{\nu(a,d)} \underbrace{\left[\frac{\partial}{\partial d} + \frac{\partial}{\partial b}\right][\tilde{d} + \Psi T - \Psi\tilde{b} - \underline{m}]}_{=0 \text{ if } \Psi=1}$$

$$= -\frac{\rho\theta(1+i)}{D} + \frac{\rho(1-\theta)(1+i)}{E} - \frac{\lambda}{\nu(a,d)}(1-\Psi) \stackrel{!}{=} 0 \qquad \text{(II)}$$

If $\Psi = 1$, i.e. if credit is used, the amount of credit $\tilde{b}$ changes one to one with payment $\tilde{d}$, and the third term in the derivative of B with respect to d vanishes (economically speaking, there is no binding cash constraint, and hence no holdup, whenever credit is used). From (II), obtain:

$$\rho(1-\theta)(1+i)D = \rho\theta(1+i)E - \lambda(1-\Psi)D^{1-\theta}E^{\theta}$$

Insert into (I) and rearrange terms to obtain the expression stated in Lemma 1.

Denote by $\nu(a)$ the optimal bargaining outcome. note that quantities q depend on a only as long as they are below their unconstrained Nash bargaining equilibrium, $q(a) < q^*$, where:

$$\tilde{q}^* = \begin{cases} \overline{q}^{*+} & \text{if } \Psi = 1\,, \overline{d} > \underline{m} \quad \text{no holdup, good match} \\ \underline{q}^* & \text{if } \Psi = 0\,, \underline{d} = \underline{m} \quad \text{holdup, bad match} \end{cases}$$

Further increases in initial endowment do not generate higher transactions in the morning market, $dq/da = 0$ if $q(a) \geq q^*$. This also means that the value of the bargaining outcome:

$$\nu(a, d) = (1 - \phi)(\nu_0) + \phi[(\tilde{u})\tilde{q}(a) - \tilde{d}(a) - \Psi T]$$

where:

$$E\nu_0 = Ed[q^*(a)] - N[q^*(a)] + U(X^*) - X^*$$

is flat in a for values of $q(a)$ that exceed the equilibrium value.

$$d\nu(a)/da = 0 \text{ if } q(a) \geq q^*$$

To see this, note that in eq. (6), for values $\tilde{q} \geq q^*$, the derivative of B^{mb} with respect to a indeed vanishes:

$$\frac{d \ln B^{mb}}{da} = \theta \frac{\rho W(1+i) - \rho W(1+i)}{D} + (1-\theta) \frac{-\rho W(1+i) + \rho W(1+i)}{E} = 0 \qquad , \tilde{q} \geq q^*$$

As q^* is unaffected, the buyer takes identical amounts of additional resources da into the afternoon market, irrespective of whether a match was established or not, leaving the solution to the Nash bargaining problem unchanged.

For $\tilde{q} \leq q^*$, $\nu(a)$ is strictly concave in a if $u'(a)$ is log-concave (see the discussion in Lagos and Wright, 2005). In the mixed cash and credit economy, each individual encounters a bad match (in which concavity of $\nu(a)$ holds) with probability $1 - \alpha$. Aggregating over all individuals, population share $1 - \alpha$ is always in a bad match, which establishes the concavity of $E\{\nu(a)\}$.

Proof of Proposition 1:

1. Under Assumption 1, money demand is at least high enough to cover cash needs in a bad match, $\tilde{m} \geq \underline{m}$. For this to hold, the value of the buyer's problem before entering the morning market with $\underline{m}$ must exceed the value of entering the morning market without cash. From eq. (3), we have the buyer's problem in a mixed cash and credit economy as:

$$\begin{aligned} V^{mb} &= \alpha[\bar{u}(\bar{q}^+) + \rho(a - \bar{d} - T + i(a - \underline{m} - \bar{b} - T))] \\ &+ (1 - \alpha)[\underline{u}(\underline{q}) + \rho(a - \underline{m} + i(a - \underline{m})] \end{aligned}$$

In a bad match, the cash constraint binds as no credit is used. Hence the transactions volume in a bad match is subject to holdup.

From eq. (2), the value of the buyer's problem in a pure credit economy is:

$$\begin{aligned} V^b &= \alpha[\bar{u}(\bar{q}^{\,+}) + \rho(a - \bar{d} - T) + i(a - \bar{d} - T)] \\ &+ (1-\alpha)[\underline{u}(\underline{q}^{+}) + \rho(a - \underline{d} - T) + i(a - \underline{d} - T)] \end{aligned}$$

Obtain $V^{mb} - V^b$ to arrive at Assumption 1. If the allocation is efficient and there is no holdup, i.e. if $\theta \in \{0,1\}$, Assumption 1 collapses into a positivity constraint on T. In either case, money demand $\tilde{m}$ under Assumption 1 is at least $\tilde{m} \geq \underline{m}$. This completes the first part of the proof.

2. Under Assumptions 1 and 2, money demand, which we know from the above to be positive and at least equal to $\underline{m}$, is actually no bigger than that, i.e. $\tilde{m} = \underline{m}$. We first have to show that under the assumptions, the value of holding $\underline{m}$ is higher than the value of a pure cash economy where $\tilde{m} = \overline{m}$. In a second step we show that money holdings $\tilde{m} > \underline{m}$ will generate lower value than W^{mb}, the value associated with holding just $\underline{m}$.

"⇒": We first observe that the trial solution $\tilde{m} = \underline{m}$ is preferred to a pure cash economy $\tilde{m} = \overline{m}$ if $V^{mb} > V^m$. From 1, the buyer's problem in a pure cash economy is:

$$\begin{aligned} V^m &= \alpha[\bar{u}(\bar{q}^{\,+}) + \rho(a - \bar{m}) + i(a - \bar{m})] \\ &+ (1-\alpha)[\underline{u}(\underline{q}^{+}) + \rho(a - \underline{m}) + i(a - \bar{m})] \end{aligned}$$

Obtain:

$$\begin{aligned} V^{mb} - V^m &= \alpha[\bar{u}(\bar{q}^{+}) + \rho(a - \bar{d} - T + i(a - \underline{m} - \bar{b} - T))] \\ &+ (1-\alpha)[\underline{u}(\underline{q}) + \rho(a - \underline{m} + i(a - \underline{m})] \\ &- \alpha[\bar{u}(\bar{q}^{\,+}) + \rho(a - \bar{d} - T + i(a - \tilde{m}^{+} - \bar{b} - T))] \\ &- (1-\alpha)[\underline{u}(\underline{q}^{+}) + \rho(a - \underline{m} + i(a - \tilde{m}^{+}))] \end{aligned}$$

This leads right back to Assumption 2, which proves the "if" part: under the assumption, a combination of cash and credit is preferred to a cash-only portfolio.

"⇐": Now assume that money holdings $\tilde{m}^*$ exceed that level, so that $\underline{m} < \tilde{m}^* < \bar{m}$. Under this strategy, agents still use credit in a good match. The value of frontloading $\tilde{m}^*$ is:

$$\begin{aligned} V^{m+} &= \alpha[\bar{u}(\bar{q}^{\,*}) + \rho(a - \bar{d} - T + i(a - \tilde{m}^* - \bar{b} - T))] \\ &+ (1-\alpha)[\underline{u}(\underline{q}^{*}) + \rho(a - \underline{m} + i(a - \tilde{m}^*)] \end{aligned}$$

However, this situation is not optimal; the cash constraint will bind in equilibrium when a bad match is realized. For a discussion see Lagos and Wright (2005, sec. 2.2).

Proof of Lemma 2:

We need to prove that money holdings $\tilde{m}$ are always bounded. From the proof of Proposition 1, money demand is finite and bounded away from zero under Assumptions 1 and 2: $\tilde{m} = \underline{m} = \underline{q} > 0$. Hence we only need to examine what happens if Assumptions 1 and 2 do not hold. As to Assumption 1, it follows immediately from the first part of the proof of Proposition 1 that $T = 0 \Leftrightarrow \tilde{m} = 0$: there is no demand for money if the use of credit is costless. This establishes the lower bound on money demand.

As to Assumption 2, Proposition 1 implies that if fixed credit cost T is high enough to violate the upper bound on T, $\alpha\rho(1+i)T \geq (1-\alpha)\rho i[\bar{m} - \underline{m}] + [\bar{u}(\bar{q}^*) - \bar{u}(\bar{q})] - [\underline{u}(\underline{q}^*) - \underline{u}(\underline{q})] \Rightarrow \tilde{m} \geq \bar{q}$. To show that the strict equality $\tilde{m} = \bar{q}$ holds, consider the value of holding cash at level $\tilde{m} = \tilde{m} = \bar{q}$, denoted before as V^m, , and compare it to the value of holding a higher amount of cash V^{m+} where $m+ > \bar{m}$. Holding this excess cash amount, agents could break out of the holdup in a good state and realize $u\bar{q}^* > u\bar{q}$. However, this would violate the optimality conditions for the cash economy of Lemma 1, and can hence be excluded.

Proof of Proposition 2:

(a) We need to show that boundedness of a and a' can be obtained by examination of the feasibility constraints. We first obtain the value function at the optimized levels, and then check its boundedness conditions. To obtain the value function, we insert the bargaining outcomes and obtain (for the mixed cash and credit economy):

$$V(a) = \max_{a'} E\{\nu(a) + \rho[a - L(a) - c - a' + \beta V(a')]\}$$

where

$$\nu(a) = \nu_0 + u[q^*(a)] - d[q^*(a)]$$

with

$$E\nu_0 = Ed[q^*(a)] - N[q^*(a)] + U(X^*) - X^*$$

and where:

$$-c = -\phi\alpha(T + i\tilde{b}) + (1-\phi)\alpha i\tilde{b}$$

As $\phi = .5$ by assumption, $c = \phi\alpha T$, which is a positive constant.

The linear terms $L(a)$ in a are obtained from (1) in the main text as:

$$0 < L(a) = \phi[(1-\Psi)\alpha\bar{m} + (1-\Psi)(1-\alpha)\underline{m}] + (1-\phi)[\alpha\Psi\tilde{b}]$$

$L(a)$ is strictly positive irrespective of whether credit is used or not, i.e. whether Ψ is equal to one or zero. It is also strictly positive if money demand is zero, in which case the demand for bonds must be positive, and vice versa. In equilibrium where $a^* = b^* + m^*$ is constant, $L(a) < 1$.

From Lemma 1 we know that $\nu(a)$ is bounded above. For the general good produced and consumed in the afternoon market, boundedness of output Y and consumption C follows from (2) in the text and from the regularity condition $U'(C^*) = 1$ for some $C^* > 0$. However, $V(a)$ is still unbounded above because of its linear terms in a. We must therefore provide upper bounds. The strategy will be to use the constraints on the endowment process and insert these into $\nu(a)$ and $V(a)$, respectively.

To find an upper bound for $V(a)$, note that Proposition 1 implied that under Assumptions 1 and 2, an upper bound for money holdings is $\underline{m} = \underline{q} > 0$. By Lemma 2, if Assumptions 1 and 2 do not hold, the upper bound for money holdings is $\bar{m} = \bar{q} > 0$. As a consequence, the transversality condition on bond holdings translates into a transversality condition on total assets:

$$\lim_{t\to\infty} \beta^t a_t = \lim_{t\to\infty} \beta^t \tilde{m} + \lim_{t\to\infty} \beta^t b_t = 0$$

where $\tilde{m}$ assumes the value $\underline{m}$ if Assumptions 1 and 2 hold, and $\bar{m}$ otherwise. Assuming a finite initial endowment value v_0, $V(a)$ must therefore be bounded. To see this, write $V_0(a)$ as:

$$\begin{aligned}
V_0(a) = \quad & E_0\{\nu_0 + u(q[a_0]) - d(q[a_0]) + a_0 - L(a_0) - a_1 - c + && \beta V(a_1)\} \\
= \quad & E_0\{\nu_0 + u(q[a_0]) - d(q[a_0]) + a_0 - L(a_0) - a_1 - c + && \\
& +\beta\ \ [\nu_1 + u(q[a_1]) - d(q[a_1]) + a_1 - L(a_1) - a_2 - c] + && \beta^2 V(a_2) \\
& \vdots \qquad \vdots && \vdots \qquad \} \\
= \quad & \textstyle\sum_{t=1}^{T} E_0\{\beta^t[\nu(a_t) + u(q[a_t]) - d(q[a_t]) + a_t - L(a_t) - a_{t+1} - c] + && \beta^T V(a_T)\}
\end{aligned}$$

Continuing the above substitution process indefinitely and taking limits:

$$V(a_0) = \lim_{T\to\infty} \sum_{t=1}^{T} E_0\{\beta^t[\nu(a_t) + u(q[a_t]) - d(q[a_t]) + a_t - L(a_t) - a_{t+1} - c] + \lim_{T\to\infty} E_0(\beta^T V(a_T)\}$$

The second limit on the RHS of this term is the transversality condition. In the first term, $\nu(a), u(a)$ and $d(a)$ are all bounded. It remains to evaluate the linear terms in a_t and a_t+1. As the sequence $\beta^t a_t$ converges to zero from above, the series generated by the partial sums of this sequence converges to a positive finite value: $\lim_{T\to\infty}\sum_{t=1}^{T} E_0 a_t = A^* > 0$. Hence, by substituting into the value function:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t E_0\{a_t - L(a_t) - a_{t+1}\} &= \sum_{t=0}^{\infty}\beta^t E_0\{a_t - L(a_t)\} + \beta^{-1}a_0 - \beta^{-1}\sum_{t=0}^{\infty}\beta^t E_0 a_t \\ &= \frac{a_0}{\beta} + \frac{\beta-1}{\beta}A^* - L(A^*)\end{aligned}$$

Both a_0 and $A*$ are finite, hence the limit exists. Then, $V(a_0)$ is bounded from above, such that there are candidate members a^* in the bounded feasible set X.

(b) Given (a), Assumptions 4.3. and 4.4. in Stokey and Lucas (1989) are met. Hence, Theorem 4.6 in Stokey and Lucas (1989) applies.

European
Historical
Economics
Society

EHES Working Paper Series

Recent EHES Working Papers

2011

EHES.1 The Role of Technology and Institutions for Growth: Danish Creameries in the Late Nineteenth Century.
Ingrid Henriksen, Markus Lampe, and Paul Sharp

EHES.2 Bondholders vs. Bond-Sellers? Investment Banks and Conditionality Lending in the London Market for Foreign Government Debt, 1815-1913.
Marc Flandreau and Juan Flores

EHES.3 The Labor Market Integration of Migrants: Barcelona, 1930.
Javier Silvestre, Mª Isabel Ayuda, and Vicente Pinilla

EHES.4 Forced Labour in Franco's Spain: Workforce Supply, Profits and Productivity
Fernando Mendiola Gonzalo

EHES.5 Was the Emergence of the International Gold Standard Expected? Melodramatic Evidence from Indian Government Securities.
Marc Flandreau and Kim Oosterlinck

All papers may be downloaded free of charge from: www.ehes.org