Arresting the Sword of Damocles: Dating the Transition to the Post-Malthusian Era in Denmark

Peter Sandholt Jensen, University of Southern Denmark

Maja Uhre Pedersen, University of Southern Denmark

Cristina Victoria Radu, University of Southern Denmark

Paul Richard Sharp, University of Southern Denmark, CAGE, CEPR
We would like to thank conference and seminar participants for helpful comments and suggestions.

Corresponding author:
Department of Business and Economics, University of Southern Denmark, Campusvej 55, 5230 Odense M, Denmark. psj@sam.sdu.dk

Arresting the Sword of Damocles: Dating the Transition to the Post-Malthusian Era in Denmark*

Peter Sandholt Jensen¹,
University of Southern Denmark

Maja Uhre Pedersen,
University of Southern Denmark

Cristina Victoria Radu,
University of Southern Denmark

Paul Richard Sharp,
University of Southern Denmark, CAGE, CEPR

Abstract

Unified Growth Theory postulates a transition from a Malthusian to a post-Malthusian era and finally to modern economic growth. Previous studies have been able to date the end of the post-Malthusian era, but none have conclusively established the timing of the end of the Malthusian era and thus transition to the post-Malthusian era. We consider the case of Denmark, which was characterized by extreme resource and environmental constraints until the final decades of the eighteenth century and thus presents a good candidate for a purely Malthusian society. We employ a cointegrated VAR model on Danish data from ca. 1733-1800, finding that evidence for diminishing returns, which characterize the “pure” Malthusian era, disappears after 1775, consistent with an increasing pace of technological progress.

JEL Codes: J1, N33, O4

Keywords: Cointegration, Denmark, Malthusian, post-Malthusian

We would like to thank conference and seminar participants for helpful comments and suggestions.
¹ Corresponding author: Department of Business and Economics, University of Southern Denmark, Campusvej 55, 5230 Odense M, Denmark. psj@sam.sdu.dk

Notice
The material presented in the EHES Working Paper Series is property of the author(s) and should be quoted as such. The views expressed in this Paper are those of the author(s) and do not necessarily represent the views of the EHES or its members.
1. Introduction

“Thanks to coal and iron, the energy and raw materials crisis was overcome, and the sword of Damocles hanging over Denmark and the whole of Europe was removed. The pessimists, who had preached that Denmark would freeze to death and that the country would have to be abandoned as uninhabitable, were put to shame... In the 1820s, the gloomy prophecies abounding in books and periodicals vanished like the morning dew. Even the memories of the energy and raw materials crisis that had threatened society with slow strangulation disappeared.”

- Kjærgaard 1994, p. 128

Malthusian concerns are nothing new, as witnessed by historical fears about shortages of land, coal, oil or more generally today about the capacity of the planet to sustain economic growth. Malthus was not even the first to raise such concerns, with another priest, from the Danish island of Lolland, Daniel Huusfeld, even claiming in 1771 that Martin Luther had prophesized the end of the world due to a lack of wood centuries earlier: “Our blessed and - in his time - perspicacious Lutherus has foreseen that a shortage of wood would lead to the end of the world, and no doubt cause great suffering to the human race, mostly towards the globe’s North Pole”. (Huusfeld 1771, p. 30, quoted by Kjærgaard, 1994). Such concerns were, however, not unfounded, since for most of human existence the world has indeed been characterized by constraints which were only truly overcome with the demographic transition and the industrial revolution. The timing of these events can to a large extent predict relative economic development today, a central message of Unified Growth Theory (UGT, Galor 2005, 2011). The present work seeks to test the validity of the transitions predicted by UGT in a setting which, as we will explain below, is uniquely fit for purpose: eighteenth century Denmark, which was experiencing an extensive ecological crisis at that time.

Malthus, starting with his 1798 essay “An Essay on the Principle of Population”, explained how the preindustrial world could be understood through three assumptions: the first is that an increase in wages determines an increase in the birth rates because people marry earlier and so they have more children (“the preventive check”); the second effect is that the increase in wages determines a decrease in death rates because people will live in better conditions (“the positive check”); the third assumption is that an increase in labor gives diminishing returns, because the amount of land that can feed the population is fixed. These assumptions combined imply that an improvement in technology, while initially leading to an increase in wages, will simply cause population to grow and wages to decrease again. According to UGT, societies escaped Malthusian stagnation through the gradual accumulation of technological knowhow prompted by the expansion of the population (so-called ‘scale effects’ - see Boserup 1965 and Kremer 1993), which in turn was made possible by the productivity advances due to technological progress.
The availability of new data in recent decades has allowed increasingly sophisticated tests on preindustrial data, which have by and large confirmed some of Malthus’ theory. Much of these analyses have considered England. For example, Møller and Sharp (2014) build on earlier work by Nicolini (2007), who first suggested that the Malthusian model should be estimated as a VAR model, and contribute to this literature by formalizing the Malthusian model as it is generally understood today and demonstrating how it can be tested within a Cointegrated Vector AutoRegressive (CVAR) framework. This approach has the advantage that it allows a focus on what can be interpreted as long run equilibrium relationships rather than the emphasis on short run effects which had previously been common in the literature (see for example Lee 1981), or on non-classical estimation techniques with only loose theoretical underpinnings (Lee and Anderson 2002, Rathke and Sarferaz 2010). Lee and Anderson (2002) used data on real wages developed in Phelps-Brown and Hopkins (1956) along with vital statistics from Wrigley and Schofield (1981) to apply the Kalman filter technique to find evidence for a preventive and positive check as well as an elasticity of the real wage with respect to the size of the population very close to -1. In his work, Nicolini (2007) has found weak positive checks that disappeared before the middle of the seventeenth century and stronger preventive checks disappearing before the middle of the eighteenth century. Similar results were found by Crafts and Mills (2009), who however used data on real wages from Clark (2007) which, they argue, represent living standards in a more accurate way. Finally, Kelly and Ó Gráda (2014) gathered data on inheritances, extending mortality estimates far back to 1250 and find that even if real wages were falling, the positive check had lost strength by 1650. They based this on the emergence of public charity in the same period, thus highlighting the role of government actions on the impact of harvest failure on mortality.

A disadvantage with the focus on England, however, has been that it was certainly an historical outlier in many ways. Important in this context is of course its early industrialization, which Møller and Sharp identify as central to the disintegration of the Malthusian mechanisms in the late eighteenth century. Thus, other scholars have recently attempted tests using data from other countries: France (Murphy 2010), Germany (Pfister and Fertig 2010), Northern Italy (Chiarini 2007, 2010; Fernihough 2013; Pedersen et al 2020). Most relevant for the present work is however the analysis by Klemp and Møller (2016). They apply Møller and Sharp’s approach (although without marriage rates) to Scandinavian data, finding results similar to those for England. For Denmark, Norway and Sweden, they find evidence for post-Malthusian dynamics until around 1900, at which point the preventive check flips sign, consistent with a quantity-quality trade off. Thus, the timing of the transition from the post-Malthusian to the modern regime has been established, but what about the transition from the Malthusian to the post-Malthusian regime?
New wage data for Denmark in the eighteenth century (Jensen et al. 2020) allows us to investigate this. Moreover, a focus on this country gives clear advantages. First, the demographic data is considered to be of superior quality, since official statistics of births and deaths were kept from the 1730s (Gille 1949), which although sometimes of dubious quality, are an order of magnitude more reliable than the samples of church records of christenings and funerals used to provide estimates of the vital rates of other countries (see for example Wrigley and Schofield 1989). More importantly, however, Denmark appears to have experienced an extreme and well-documented Malthusian crisis in the run up to the eighteenth century, characterized, among other things, by the overexploitation of the land and an acute shortage of wood for fuel and other uses (Kjærgaard 1994). This was only solved by a variety of measures around the turn of the nineteenth century. Thus, our case provides a perfect testing ground for UGT, and allows us to test for the first of the transitions proposed by UGT, and we do so by applying the statistical framework proposed by Møller and Sharp (2014). This has the obvious advantage that our results can be directly compared to those of Møller and Sharp (2014), as well as with those of Klemp and Møller (2016). We do indeed find strong evidence for a transition from a Malthusian to a post-Malthusian society in around the 1770s in Denmark. Combining this with the evidence presented by Klemp and Møller (2016), we can thus conclude that Denmark was Malthusian until the late eighteenth century, then post-Malthusian for a little over a century, and was enjoying modern economic growth from the late nineteenth century.

Our findings have importance for the considerable body of literature which takes inspiration from UGT and seeks to understand the transition from stagnation to growth, the differential timing of this around the world, and thus the massive difference in living standards between countries we observe today. Fundamental to this literature is the interaction between human evolution and economic development (see also the survey by Ashraf and Galor 2018): the Malthusian preventive check, by implying greater fertility of more economically successful individuals, meant that their share of the population gradually increased. This success might be due to innate (genetic) preferences for the quality rather than quantity of offspring, resistance to infectious diseases, human body size, time preference, and more (see for example Galor and Moav 2002, 2007, Lagerlöf 2015, Galor and Özak 2016). Beyond the evidence for Malthusian and post-Malthusian regimes we contribute to here, there is in fact growing empirical support for UGT. For example, Ashraf and Galor (2011) have found evidence that population density was determined by productivity, and Galor and Klemp (2019) using historical population registers from Quebec, have offered convincing evidence for changes in preference regarding quality and quality of children.

The remainder of this work proceeds as follows. In the next section, we present the historical background and explain the Malthusian pressures which Denmark was experiencing in the eighteenth century, and how
these were relieved. In Section 3, we summarize the theoretical underpinnings of our empirical analysis, which we turn to in Section 4. Section 5 concludes.

2. Historical Background

According to Kjærgaard’s (1994) ecological account of the history of Denmark, before the Black Death, Danes lived “in a state of misery without parallel in the country’s history” (quoted by Kjærgaard 1994, from Helge Paludan 1977, pp. 412, 414—15). However, in true Malthusian fashion, and in common with much of the rest of Europe, living standards peaked after the population decline precipitated by the pandemic. Historical accounts mention deserted farms and general shortage of population in mid-fourteenth century Denmark3, combined with a recovery of the ecosystem during the fifteenth and sixteenth centuries, as forests expanded, moorlands contracted, and sand dunes stabilized. Then, of course, the population began to recover. In 1650 it was around 550,000, probably around half that in 1300. By 1735 it had grown to 715,000, in 1774 to 815,000, and by 1800 it was 925,000, and was thus back to where it had been 500 years earlier.

Alongside this population growth, continuous wars with Sweden meant that there was enormous pressure on resources from the growth of the fiscal-military state on the basis of taxation in seventeenth and eighteenth centuries. This showed itself both through civilian and military expenditure. For example, the construction of Christiansborg Palace in 1731-45 required ten thousand beech trunks for its foundations alone, and the firing of around thirty million bricks required close to ten thousand tons of wood as fuel. This was an extreme example, of course, but timber and bricks were also in demand for the construction of for example lighthouses and other purposes, although this was moderate compared to the rest of Europe. What was not, however, was consumption for military purposes. During the seventeenth century, Denmark became the strongest militarized nation in Europe, with the building of naval vessels playing a central role. This, supplemented by consumption for ground forces, used vast quantities of timber and led to massive ecological destruction. Moreover, this was funded by extremely high taxes on the countryside which led to increased agricultural production and an overproduction crisis.

According to Kjærgaard, this Malthusian crisis showed itself in four ways. First, through the devastation of the forests. Around 1600, 20-25 percent of Denmark was covered by forest. By 1750 this had fallen to only

---

3 We refer to Denmark as being the “Kingdom of Denmark”, which did not include for example Greenland, Iceland, Norway and the Duchies of Schleswig and Holstein. Northern Schleswig constitutes part of modern Denmark today, but otherwise the “Kingdom of Denmark” was similar in geographical extent to modern Denmark.
8-10 percent, a catastrophe for an economy in which wood was fundamental, as fuel in households and all kinds of industries, and as timber for the construction of ships, houses, and fences. Alongside this, forest hunting disappeared. Second, sand and mould drift became an increasing problem. This began in the sixteenth century, and by the seventeenth and eighteenth centuries, dunes were spreading deep into the countryside. Forests and fertile arable land were transformed into “sandy desserts or windswept surfaces almost devoid of topsoil” due to overexploitation by forestry and agriculture so that for example by 1750 as much as five percent of Jutland was no longer cultivated due to sand and mould drift. Contemporaries had to suffer through the massive sandstorms which this precipitated. Third, the water level was affected. Tree felling and soil erosion led to more flooding, acidification of the soil, increased peat formation, and a shortened growing season. Finally, nitrogen was at a minimum practically everywhere. This meant that production was low and what was even worse unsustainable. It was not even possible to rely on a stable supply of manure for fertilizing the land: both animal fodder and manure was burned as fuel due to lack of wood. The lack of fodder meant that livestock was malnourished, meaning that when cattle plague struck in the 1740s, around half the national cattle herd, around 250,000 head, were killed in just a few years.

Kjærgaard argues that crisis was however averted by a number of innovations. The central administration only took measures to stop sand drift in the late 1700s, making use of a massive conscription of men and vehicles, with a network of officials appointed to oversee the work. Once the formation of deserts was halted, new land needed to be reclaimed through damming, draining, marling, and from the sea. There were also major developments in agriculture, see Jensen et al (2018) and Lampe and Sharp (2018). These included new crop rotation techniques, increased efforts to retain manure, with the spread of stall feeding, and the introduction of domesticated clover (see Schmidt et al 2018). The end result was that agricultural production became greater than ever before. As for fuel, first, various ways were found to economize on wood consumption. For example, new building materials were used, and more efficient means of heating were developed. Wooden fences were replaced by stone walls and hedges, or by turf or seaweed. Ways were found to reduce the use of timber for shipbuilding, and stone was used for surfacing roads instead of wood. Second, laws were passed to protect the forests. Third, peat, waterpower and wind power were increasingly used for fuel. Finally, there was an increased use of imported wood substitutes: iron and coal. Kjærgaard somewhat controversially argues that the driving forces behind what he terms the “Danish Revolution” were not the agrarian reforms of the late 1700s, which he claims were mostly about the redistribution of political power within the elite. Rather, it was the consequence of growing technological potential through science and the printed word. Whatever the case, the end result was that, as stated in the opening quote, the ecological and Malthusian crisis was soon forgotten. It remains, however, to be shown whether the data
supports an end to the Malthusian era at this point – which is what the remainder of the present work seeks to test.

3. The Theoretical Model

Before turning to the empirical analysis, we first recap the basic Malthusian model as formalized by Møller and Sharp (2014). The model assumes a closed economy with no trade and migration, fixed land supply, labor supply to be proportional to total population, fully flexible prices, and supply determined output. The Malthusian system as described above can be characterized by the following system of five equations:

\[ w_t = c_0 - c_1 \ln N_t + \ln A_t \]  
\[ b_t = a_0 + a_1 w_t + \varepsilon_{bt} \]  
\[ d_t = a_2 - a_3 w_t + \varepsilon_{dt} \]  
\[ \ln A_t = \ln A_{t-1} + \varepsilon_{At} \]  
\[ \ln N_t \equiv \ln N_{t-1} + b_{t-1} - d_{t-1} \]

Equation (1) describes the relationship between real wages and population growth, where \( w_t \) represents the real wage, \( c_0 \) the intercept, and \( N_t \) the total population size. The minus in front of \( c_1 \) reflects diminishing marginal returns and finally \( A_t \) represents technology/arable land or capital per worker. Equation (2) describes the preventive check, where real wages are expected to affect birth rates positively. The parameter \( b_t \) represents the birth rate, and \( a_0 \) is the intercept. \( a_1 \) captures the preventive check itself, and \( \varepsilon_{bt} \) is a stochastic shock, with mean zero, and constant variance. The stochastic shock also represents unsystematic influences on births, not explained by the model. Equation (3) describes the positive check, where the death rate is expected to be negatively affected by increases in real wages. \( d_t \) represents the death rate, \( a_2 \) is the intercept, \(-a_3\) captures the positive check, and \( \varepsilon_{dt} \) is a stochastic shock, with mean zero, and constant variance which also represents unsystematic influences on deaths which are not captured by the model. The rationale behind the two checks has been discussed extensively in the literature. The positive check can be motivated through biology and the impact of low incomes on nutrition and infant mortality. For the preventive check more rational economic behavior has been invoked – see for example Cinnirella et al (2017).
Technology is described by equation (4) and is modeled as a non-stationary stochastic process, specifically a random walk (Møller and Sharp 2014). It is exogenous and persistent, with $A_t$ representing the aggregate level of technology, and $\varepsilon_{dt}$ a stochastic shock with mean zero and constant variance. This latter also represents unsystematic influences on technology which cannot be explained by the model. Finally, equation (5) describes population growth (or labor supply). $b_{t-1} - d_{t-1}$ represents the difference between the birth rate and the death rate (the rate of population growth) which in turn is determined by the previous equations.

The short-run equilibrium results from solving the above system of equations. When technology $A_t$ is held fixed, population growth $b_{t-1} - d_{t-1} = 0$ and there are no shocks, the steady state values of the economy can be illustrated as in Figure 1. Here, the short-run equilibrium is represented by the dashed lines, while the steady state, given $A_t$, is represented with arrows for period $t$.

![Figure 1: The Malthusian model, based on Møller and Sharp (2014) and Equations (1)-(5)](image)

**Note:** The preventive check relation (upward sloping) and the positive check relation (downward sloping) are presented in the left panel. The downward sloping labor demand and vertical labor supply are presented in the right panel. The short-run equilibrium is shown by the dotted lines, while the steady state, given $A_t$, is represented by arrows.

The steady state values of real wages, birth rate and death rate ($w^*, b^*, d^*$) do not depend on time while the steady state level of population, $\ln N^*_t$, depends on $A_t$. In the right panel it is apparent that the short-run equilibrium level of real wages is determined by the stocks of population and technology. In the left panel the birth and death rates are determined adding their respective shocks, $\varepsilon_{b,t}$ and $\varepsilon_{d,t}$. In the absence of
further shocks all variables will converge to their steady-state values in the subsequent periods, \{w^*, \ln N_t^*, b^*, d^*\}. Exactly how to test for a post-Malthusian regime is discussed in the following section, but intuitively \(c_1\) should econometrically be found to be insignificant, reflecting the fact that technological progress offsets the income reducing potential of diminishing returns.

4. The Data and the Econometric Approach

We follow Klemp and Møller (2016) in using Gille’s (1949) data on vital rates.\(^4\) For the real wages, we rely on news series for unskilled day laborers constructed by Khaustova and Sharp (2015) for the urban sector and Jensen et al (2020) for the rural sector. These are illustrated in Figure 2.

![Figure 2: Crude birth and death rates and real wage indices (=1 in 1800) for Denmark, 1705-1800](image)


\(^{4}\) This unfortunately precludes using marriage rates as in Møller and Sharp (2014), but these do not seem to be available for Denmark.
We then proceed to follow Møller and Sharp (2014), which can be consulted for a more detailed description of the method\(^5\), and perform the analysis as follows. First, we estimate an unrestricted VAR model. Second, we analyze the residuals in order to be certain that the model is well specified, i.e. that the errors do not exhibit heteroskedasticity or autocorrelation and are normally distributed. Third, if the unrestricted VAR model is well-specified, then we test for non-stationarity and for the cointegration rank. Dummies can be used to account for different types of outliers before testing for stationarity. Finally, if the rank is 2, we can apply the co-integrated VAR model and then continue with robustness checks.

The unrestricted VAR model is written as:

\[
\Delta x_t = \Pi x_{t-1} + \Gamma_1 \Delta x_{t-1} + \cdots + \Gamma_{k-1} \Delta x_{t-(k-1)} + \Phi D_t + e_t, \tag{6}
\]

where \(x_t' = (w_t, b_t, d_t)\), \(k\) represents the lags, \(e_t \sim \text{Niid}(0, \Omega)\) and \(\Phi D_t\) represents the trend. The matrix \(\Pi\) contains the coefficients of interest and can be found by solving the system of equations (1)-(5) with respect to the three observed variables. If \(\text{det}(\Pi) \neq 0\) then \(x_t\) is stationary. Otherwise, if \(\text{det}(\Pi) = 0\) it means that \(x_t\) is non-stationary and \(\Pi\) has reduced rank, \(r < 3\). We can decompose the matrix \(\Pi\) in the following way

\[
\Pi = \alpha \beta', \tag{7}
\]

where \(\alpha\) and \(\beta\) are \(3 \times r\) matrices with \(r < 3\). Note that \(\beta' x_t \sim I(0)\) is a stationary relation given that the variables are cointegrated. \(\beta\) represents the cointegration coefficients, \(\alpha\) represents the error correction coefficients (the so-called adjustment coefficients) which tell us which of the variables are error correcting whenever the system is out of equilibrium.

The determinant of \(\Pi\) is given by the equation:

\[
\text{det}(\Pi) = -\bar{c}_1 (a_1 + a_3) \tag{8}
\]

From equation (8) it is clear that the determinant will equal zero in one of two cases:

1) \(a_1 + a_3 = 0\) or equivalently \(a_1 = -a_3\). This hypothesis indicates that the population is independent of income, because the effect on births is the same as on deaths. The restrictions on \(\alpha\) and \(\beta'\) become:

---

\(^5\) See also Juselius (2006) and Møller (2008).
In $\beta'$ the first row implies a stationary growth rate of population and the second is a ‘check relation’. In this case we also observe $c_1$, diminishing returns, and its significance implies homeostasis, i.e. that any temporary increases in income for example due to a new technology will eventually be reversed.

2) $\bar{c}_1 = 0$, which is the post-Malthusian hypothesis stating that wages are no longer dependent on vital rates. Empirically this can be tested by imposing the following restrictions on $\alpha$ and $\beta'$:

$$\alpha = \begin{pmatrix} -c_1 & 0 \\ a_3 c_1 & -1 \\ 1 + a_3 c_1 & -1 \end{pmatrix} \quad \beta' = \begin{pmatrix} 0 & 1 & -1 \\ a_3 & 1 & 0 \end{pmatrix}$$  \hspace{1cm} (9)

In $\beta'$ it is possible to read the two parameters of the preventive and the positive check directly.

Thus, empirically, we can impose the restrictions described in equations (9) and (10) to test whether a reduced rank is consistent with one or both of the equations. Before doing so, however, it is first necessary to test the rank of the matrix $\Pi$ using our two measures for real wages and the vital rates, and we thus proceed by testing the hypothesis of reduced rank, $H(2)$, against the hypothesis of full rank, $H(3)$. Reduced rank implies that we can proceed with the analysis by imposing the restrictions from equations (9) and (10). Full rank, on the other hand, implies a stationary system and the full Malthusian model.

The above account, however, might lead us to expect a transition from a Malthusian regime to a post-Malthusian regime at some point towards the end of the 1700s, when technological progress began to take off, and an environmental catastrophe was averted. Thus, in Figure 3 we illustrate the results of recursively calculated $p$-values for the test of full rank, where the end point is held fixed, but the starting point moves. Higher $p$-values indicate that we cannot reject the hypothesis of reduced rank. Both for urban and rural wages we do indeed find a quite clear break at the beginning of the 1760s. Thus, we can conclude from the results of the trace test analysis that our sample should be split into two subperiods: the first Malthusian and the second possibly consistent with equation (9) or (10).
Figure 3: The trace test calculated recursively

Note: The top panel presents the p-values for the recursively calculated trace test for $H(2)$ against $H(3)$ using urban wages. The bottom panel presents the p-values for the recursively calculated trace test for $H(2)$ against $H(3)$ using rural wages. The horizontal lines represent the 10% significance level. The end point of the sample is fixed to 1800.

We thus proceed to split the sample at the year 1775, and analyze the periods 1733-75 and 1775-1800 for both rural and urban workers. Clearly, the choice of the year 1775 might be important for the analysis, and this is explored more carefully in Appendix B. For the sake of comparability with earlier work we focus here on the results using urban wages, but those using rural wages are given in Appendix C.

Tables 1 and 2 show estimates for urban workers from 1733 to 1775. The results are much more consistent with homeostasis (equation 9) rather than the post-Malthusian hypothesis (equation 10), $p = 51\%$ vs. 7%. We find an estimate of $\hat{c}_1 = 0.006$, implying slow but significant diminishing returns. By contrast, Tables 3 and 4

---

6 The same analysis for the whole sample is in Appendix A. The results are largely insignificant.
repeat this exercise for the period 1775-1800. Now, although we cannot reject either hypothesis, the post-Malthusian explanation enjoys a much higher $p$-value (31% vs. 14%). Moreover, we now see much clearer evidence for the preventive check with a similar magnitude in both specifications ($a_1 = 22$ and $a_1 = 25$) – again consistent with a post-Malthusian regime. For rural workers, the results in Appendix C reveal even stronger support for the post-Malthusian hypothesis: $p$-values of 9% vs. 43%.

It is useful to compare the results in Table 4 with those found by Klemp and Møller (2016) for Denmark and the period 1824-90. They find both checks to be significant with $a_1 = 3.9$ and $a_3 = 5.7$, although the elasticity of the preventive check and the adjustment coefficients are smaller, at roughly half those in Table 4. This implies a weaker preventive check, and indeed, after around 1890, they find that evidence that the check mechanisms breaks down as Denmark moved towards a modern growth regime. Based on the evidence here, we can add to this conclusion: the post-Malthusian era began in Denmark around 1770, and lasted for a little over a century.

---

7 An obvious question would be why we do not merge their data with ours, and estimate the whole post-Malthusian era at once. The problem is that the Danish economy was destroyed by the Napoleonic Wars, and the currency collapsed, meaning that there are no meaningful real wages from around 1800 and into the 1820s, when a new currency was introduced.
### Table 1: Testing for homeostasis (equation 9) for urban workers (1733-1775)

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\beta^i$</th>
<th>$w_t$</th>
<th>$b_t$</th>
<th>$d_t$</th>
<th>Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta w_t$</td>
<td>$-0.006$</td>
<td>$0.000$</td>
<td>$\beta_1^i$</td>
<td>$-0.000$</td>
<td>$1.000$</td>
<td>$-1.000$</td>
</tr>
<tr>
<td></td>
<td>$(0.002)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta b_t$</td>
<td>$-0.073$</td>
<td>$-0.974$</td>
<td>$\beta_2^i$</td>
<td>$0.670$</td>
<td>$1.000$</td>
<td>$0.000$</td>
</tr>
<tr>
<td></td>
<td>$(0.070)$</td>
<td>$(0.240)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta d_t$</td>
<td>$0.798$</td>
<td>$-0.713$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(0.192)$</td>
<td>$(0.652)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of obs.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LR test of identifying restrictions $\chi^2(2)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Standard errors are in parenthesis while $p$-value of the LR-test is in square brackets. Coefficients significantly different from zero at the 5% level are in bold.

### Table 2: Testing for the post-Malthusian hypothesis (equation 10) for urban workers (1733-1775)

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\beta^i$</th>
<th>$w_t$</th>
<th>$b_t$</th>
<th>$d_t$</th>
<th>Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta w_t$</td>
<td>$-0.000$</td>
<td>$-0.000$</td>
<td>$\beta_1^i$</td>
<td>$-1.242$</td>
<td>$1.000$</td>
<td>$0.000$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta b_t$</td>
<td>$-0.886$</td>
<td>$-0.000$</td>
<td>$\beta_2^i$</td>
<td>$8.623$</td>
<td>$0.000$</td>
<td>$1.000$</td>
</tr>
<tr>
<td></td>
<td>$(0.180)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta d_t$</td>
<td>$-0.825$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(0.177)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of obs.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LR test of identifying restrictions $\chi^2(4)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Standard errors are in parenthesis while $p$-value of the LR-test is in square brackets. Coefficients significantly different from zero at the 5% level are in bold.
Table 3: Testing for homeostasis (equation 9) for urban workers (1775-1800)

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta'$</th>
<th>( w_t )</th>
<th>( b_t )</th>
<th>( d_t )</th>
<th>Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta w_t$</td>
<td>$-0.002$</td>
<td>$0.000$</td>
<td>$\beta'_1$</td>
<td>$0.000$</td>
<td>$1.000$</td>
</tr>
<tr>
<td>(0.004)</td>
<td></td>
<td></td>
<td>(4.301)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta b_t$</td>
<td>$0.029$</td>
<td>$-0.911$</td>
<td>$-21.751$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.083)</td>
<td>(0.243)</td>
<td>(3.498)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta d_t$</td>
<td>$0.715$</td>
<td>$-1.288$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.214)</td>
<td>(0.641)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Number of obs. | 26 |

LR test of identifying restrictions $\chi^2(2)$ | 3.942 | [0.1394] |

Note: Standard errors are in parenthesis while $p$-value of the LR-test is in square brackets. Coefficients significantly different from zero at the 5% level are in bold.

Table 4: Testing for the post-Malthusian hypothesis (equation 10) for urban workers (1775-1800)

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta'$</th>
<th>( w_t )</th>
<th>( b_t )</th>
<th>( d_t )</th>
<th>Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta w_t$</td>
<td>$-0.000$</td>
<td>$-0.000$</td>
<td>$\beta'_1$</td>
<td>$1.000$</td>
<td>$0.000$</td>
</tr>
<tr>
<td>(0.217)</td>
<td>(4.301)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta b_t$</td>
<td>$-0.829$</td>
<td>$-0.000$</td>
<td>$\beta'_2$</td>
<td>$-9.496$</td>
<td></td>
</tr>
<tr>
<td>(0.217)</td>
<td>(12.494)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta d_t$</td>
<td>$0.000$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.251)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Number of obs. | 26 |

LR test of identifying restrictions $\chi^2(4)$ | 4.785 | [0.3101] |

Note: Standard errors are in parenthesis while $p$-value of the LR-test is in square brackets. Coefficients significantly different from zero at the 5% level are in bold.
5. Conclusion

Denmark faced massive and well-documented Malthusian constraints by the eighteenth century, which were solved by massive investments in new technologies and practices. This makes it an ideal testing ground for identifying the onset of the post-Malthusian regime as postulated by UGT – something which assuredly happened far earlier in better developed and well-documented countries such as England.

Using evidence from both the urban and rural sectors of the economy, we find striking evidence for a transition from a Malthusian to a post-Malthusian regime in around the 1770s, consistent with the historical evidence. Combining this with a similar analysis for the nineteenth century, we can conclude that Denmark was post-Malthusian for a little over a century, when the demographic transition and rapid industrialization of the economy led to an onset of modern economic growth (see for example Henriksen 1993 and Lampe and Sharp 2018).
References


Appendix A: Regression results of the full sample 1733-1800

Tables A1 and A2 perform the same analysis as in Tables 1-4, but for the entire sample (1733-1800), using the wages of unskilled urban workers. Table A1 reveals that it is not possible to reject the homeostasis hypothesis, given a $p$-value of 11%, but the check coefficient is barely significant. As regards the post-Malthusian hypothesis, Table A2 reveals it to be strongly rejected. The recursive exercises in Appendix B reveal these results to be highly dependent on the sample chosen, however, again motivating our decision to split the data in 1775.
Table A1: Testing for homeostasis (equation 9) for unskilled urban workers (1733-1800)

<table>
<thead>
<tr>
<th></th>
<th>( \alpha )</th>
<th>( \beta' )</th>
<th>( w_t )</th>
<th>( b_t )</th>
<th>( d_t )</th>
<th>Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta w_t )</td>
<td>(-0.004)</td>
<td>0.000</td>
<td>(-0.000)</td>
<td>1.000</td>
<td>(-1.000)</td>
<td>(-0.031)</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta b_t )</td>
<td>(-0.019)</td>
<td>(-0.653)</td>
<td>(-4.404)</td>
<td>1.000</td>
<td>0.000</td>
<td>(-0.003)</td>
</tr>
<tr>
<td></td>
<td>(0.049)</td>
<td>(0.160)</td>
<td>(2.457)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta d_t )</td>
<td>(0.674)</td>
<td>(-0.804)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.131)</td>
<td>(0.423)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Number of obs. 68

LR test of identifying \(4.454\)

Restrictions \(\chi^2(2)\) [0.1079]

Note: Standard errors are in parenthesis while \(p\)-value of the LR-test is in square brackets. Coefficients significantly different from zero at the 5% level are in bold.

Table A2: Testing the post-Malthusian hypothesis (equation 10) for unskilled urban workers (1733-1800)

<table>
<thead>
<tr>
<th></th>
<th>( \alpha )</th>
<th>( \beta' )</th>
<th>( w_t )</th>
<th>( b_t )</th>
<th>( d_t )</th>
<th>Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta w_t )</td>
<td>(-0.000)</td>
<td>(-0.000)</td>
<td>(\beta_1')</td>
<td>1.000</td>
<td>0.000</td>
<td>(-0.002)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(3.160)</td>
<td></td>
<td></td>
<td>(0.011)</td>
</tr>
<tr>
<td>( \Delta b_t )</td>
<td>(-0.612)</td>
<td>(-0.000)</td>
<td>(\beta_2')</td>
<td>(-6.197)</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>(0.125)</td>
<td></td>
<td></td>
<td>(8.249)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta d_t )</td>
<td>(-0.000)</td>
<td>(\beta_3')</td>
<td>(-0.620)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.118)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Number of obs. 68

LR test of identifying 12.868

Restrictions \(\chi^2(4)\) [0.0119]

Note: Standard errors are in parenthesis while \(p\)-value of the LR-test is in square brackets. Coefficients significantly different from zero at the 5% level are in bold.
Appendix B: Recursive Analysis

In the following we present the results of a forward recursive analysis to show how our results are influenced by the selection of the ending point and how they change over time, for both urban and rural wages. The start year is always 1733, and then we gradually expand the sample by letting the end year change and plot the results. The CVAR model relies on constant parameters and is thus invalidated by the results below. We argue that splitting the sample in 1775 is consistent both with the empirical and with the historical evidence.

Figure B1: Recursive estimates of the check relation for urban workers for the specification considering homeostasis (1733-1800).
Notes: The point estimate of the preventive check coefficient, stated as $a_1$, together with the 2x Standard Error limits. Sample start is fixed at 1733.
Figure B2: Recursively estimated $p$-value for urban workers for the specification considering homeostasis (1733-1800).

Notes: $p$-values of the test statistic corresponding to the homeostasis hypothesis, where values below the dashed line indicate a rejection of the hypothesis at the 5 percent significance level.
Figure B3: Recursive estimates for urban workers for the specification considering the post Malthusian hypothesis (1733-1800).

Notes: (top panel) the point estimate of the preventive check coefficient, stated as $\alpha_1$, together with the 2x Standard Error limits; (bottom panel) point estimate of the positive check coefficient stated as $\alpha_3$, together with the 2x Standard Error limits. Sample start is fixed at 1733.
Figure B4: Recursively estimated $p$-value for urban workers for the specification of the post Malthusian hypothesis (1733-1800).

Notes: $p$-values of the test statistic corresponding to the homeostasis hypothesis, where values below the dashed line indicate a rejection of the hypothesis at the 5 percent significance level.
Figure B5: Recursive estimates of the check relation for rural workers for the specification considering homeostasis (1733-1800).

Notes: The point estimate of the preventive check coefficient, stated as $a_1$, together with the $2\sigma$ Standard Error limits. Sample start is fixed at 1733.
Figure B6: Recursively estimated $p$-value for rural workers for the specification considering homeostasis (1733-1800). *Notes:* $p$-values of the test statistic corresponding to the homeostasis hypothesis, where values below the dashed line indicate a rejection of the hypothesis at the 5 percent significance level.
Figure B7: Recursive estimates for rural workers for the specification considering the post Malthusian hypothesis (1733-1800).

Notes: (top panel) the point estimate of the preventive check coefficient, stated as $a_1$, together with the 2x Standard Error limits; (bottom panel) point estimate of the positive check coefficient stated as $a_3$, together with the 2x Standard Error limits. Sample start is fixed at 1733.
Figure B8: Recursively estimated $p$-value for urban workers for the specification of the post Malthusian hypothesis (1733-1800).

Notes: $p$-values of the test statistic corresponding to the homeostasis hypothesis, where values below the dashed line indicate a rejection of the hypothesis at the 5 percent significance level.
Appendix C: Results Using Rural Wages

In Tables C1-4 we repeat the analyses in Tables 1-4 but for rural workers. Although the results from before 1775 are somewhat unclear, and might imply that a stationary VAR would be more appropriate for a possibly more purely Malthusian economy, for the period after 1775, the post-Malthusian hypothesis can again be accepted with a high $p$-value of 43% and with a very strong and highly significant preventive check.
Table C1: Testing for homeostasis (equation 9) for rural workers (1733-1775)

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>(\beta')</th>
<th>(w_z)</th>
<th>(b_z)</th>
<th>(d_z)</th>
<th>Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta w_z)</td>
<td>0.006</td>
<td>0.000</td>
<td>(\beta'_1) 0.000</td>
<td>1.000</td>
<td>-1.000</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Delta b_z)</td>
<td>0.060</td>
<td>(-1.049)</td>
<td>(\beta'_2) 0.052</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.074)</td>
<td>(0.252)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Delta d_z)</td>
<td>(0.824)</td>
<td>-0.436</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.204)</td>
<td>(0.698)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Number of obs. 43

LR test of identifying restrictions \(\chi^2(2)\) 7.962 [0.0187]

Note: Standard errors are in parenthesis while p-value of the LR-test is in square brackets. Coefficients significantly different from zero at the 5% level are in bold.

Table C2: Testing the post-Malthusian hypothesis (equation 10) for rural workers (1733-1775)

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>(\beta')</th>
<th>(w_z)</th>
<th>(b_z)</th>
<th>(d_z)</th>
<th>Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta w_z)</td>
<td>0.000</td>
<td>0.000</td>
<td>(\beta'_1) 0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(2.017)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Delta b_z)</td>
<td>(-0.896)</td>
<td>9.421</td>
<td>(\beta'_2) 0.000</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>(0.181)</td>
<td>(5.050)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Delta d_z)</td>
<td>0.000</td>
<td>-0.976</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.182)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Number of obs. 43

LR test of identifying restrictions \(\chi^2(4)\) 10.212 [0.0370]

Note: Standard errors are in parenthesis while p-value of the LR-test is in square brackets. Coefficients significantly different from zero at the 5% level are in bold.
Table C3: Testing for homeostasis (equation 9) for rural workers (1775-1800)

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta^i$</th>
<th>$w_t$</th>
<th>$b_t$</th>
<th>$d_t$</th>
<th>Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta w_t$</td>
<td>$-0.001$</td>
<td>$0.00$</td>
<td>$\beta_1^i$</td>
<td>$1.00$</td>
<td>$-1.00$</td>
</tr>
<tr>
<td>(0.005)</td>
<td></td>
<td></td>
<td>(0.114)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta b_t$</td>
<td>$0.072$</td>
<td>$-\mathbf{1.029}$</td>
<td>$\beta_2^i$</td>
<td>$1.00$</td>
<td>$0.00$</td>
</tr>
<tr>
<td>(0.088)</td>
<td>(0.250)</td>
<td>(2.456)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta d_t$</td>
<td>$0.649$</td>
<td>$-0.616$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.237)</td>
<td>(0.682)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of obs.</td>
<td>26</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$LR$ test of identifying restrictions $\chi^2(2) = \mathbf{4.902}$ [0.0862]

Note: Standard errors are in parenthesis while $p$-value of the $LR$-test is in square brackets. Coefficients significantly different from zero at the 5% level are in bold.

Table C4: Testing the post-Malthusian hypothesis (equation 10) for rural workers (1775-1800)

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta^i$</th>
<th>$w_t$</th>
<th>$b_t$</th>
<th>$d_t$</th>
<th>Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta w_t$</td>
<td>$-0.000$</td>
<td>$-0.000$</td>
<td>$\beta_1^i$</td>
<td>$1.000$</td>
<td>$0.000$</td>
</tr>
<tr>
<td>(2.855)</td>
<td></td>
<td>(0.026)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta b_t$</td>
<td>$-0.938$</td>
<td>$-0.000$</td>
<td>$\beta_2^i$</td>
<td>$-0.750$</td>
<td>$0.000$</td>
</tr>
<tr>
<td>(0.221)</td>
<td></td>
<td>(8.112)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta d_t$</td>
<td>$-0.000$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.282)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of obs.</td>
<td>26</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$LR$ test of identifying restrictions $\chi^2(4) = \mathbf{3.843}$ [0.4276]

Note: Standard errors are in parenthesis while $p$-value of the $LR$-test is in square brackets. Coefficients significantly different from zero at the 5% level are in bold.
Recent EHES Working Papers

2020

EHES 181 Ad maiorem Dei gloriam. Numeracy levels in the Guarani Jesuit missions, Éric Gómez-i-Aznar

EHES 180 Standards of Living and Skill Premia in Eighteenth Century Denmark: What can we learn from a large microlevel wage database? Peter Sandholt Jensen, Cristina Victoria Radu, Paul Sharp


EHES 178 Winners and Losers from Enclosure: Evidence from Danish Land Inequality 1682-1895, Nina Boberg-Fazlic, Markus Lampe, Pablo Martinelli Lasheras, Paul Sharp

EHES 177 Growth Recurring in Preindustrial Spain: Half a Millennium Perspective, Leandro Prados de la Escosura, Carlos Álvarez-Nogal, Carlos Santiago-Caballero

EHES 176 A “Silent Revolution”: school reforms and Italy’s educational gender gap in the Liberal Age (1861-1921), Gabriele Cappelli, Michelangelo Vasta

EHES 175 The rise of coffee in the Brazilian southeast: tariffs and foreign market potential, 1827-40, Christopher David Absell

2019

EHES 174 American Precious Metals and their Consequences for Early Modern Europe, Nuno Palma

EHES 173 The uneven transition towards universal literacy in Spain, 1860-1930, Francisco J. Beltrán Tapia, Alfonso Diez-Minguela, Julio Martínez-Galarraga, Daniel A. Tirado

All papers may be downloaded free of charge from: www.ehes.org

The European Historical Economics Society is concerned with advancing education in European economic history through study of European economies and economic history. The society is registered with the Charity Commissioners of England and Wales number: 1052680